A Unified Theory of Corporate Leverage, Intangibility, Wage, Tax, and Innovation

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Abstract

We propose a model of corporate leverage, intangibility, wage, tax, and innovation for Nasdaq firms. In the model, firms choose leverage, invest in R&D, absorb R&D-driven innovation benefit through product pricing, and pay wages determined by the value-sharing rule between inventors and shareholders. This model endogenizes the consequences of debt use for available R&D benefit, which generate a feedback effect on corporate leverage and financing inventive human capital. The model highlights the importance of innovation for corporate decisions. Our three main results are: (i) Corporate innovation discourages debt use; (ii) the relationship between leverage and inventors' wage may be positive, inverse, or non-monotonic; (iii) the concavity of marginal R&D-value loss due to debt use makes firms react more to tax rises than to tax cuts when readjusting towards the target leverage ratio. We further extend this model in human capital risk due to innovation strategy switches and inventor mobility.

Keywords: Leverage; Wage; R&D; Innovation Strategy; Inventor Mobility.

JEL classification: G32; J31; O32; J60; O31.

1. Introduction

When firms project corporate innovations, they face complex and closely intertwined leverage, intangibility, wage, tax, and innovation related decisions. How to formalize their interactions in an integrated model remains undetermined. Research questions such as how innovation affects leverage, inventors' wage, and leverage-tax sensitivity have been explored separately[1] but whether the existing answers can coexist in a framework remains unclear.

The goal of our study is to propose a tractable structural model of corporate leverage with innovations, and then integrates it into the capital structure trade-off framework—as illustrated in Figure 1—in which the intertwined consequences of debt use for available R&D benefit are characterized simultaneously for a representative firm[2,3]. By bringing debt and R&D into a unified framework, we show how they interact with each other and shape other firm characteristics.

Our baseline model only requires the essential building blocks below: (i) the debt contract has a covenant specifying that the firm is forced into bankruptcy if its after-tax asset liquidation value falls below the debt principal as in Leland (1994); (ii) a fixed value-sharing rule between inventors and shareholders like He (2011) and Michaels et al. (2019); and (iii) all intangible assets are lost in liquidation. This parsimonious model already simultaneously captures the capital structure trade-off theories with three cross-sectional puzzling leverage features: (i) almost-zero-leverage anomaly; (ii) mixed leverage-wage relationship; and (iii) asymmetric

[1] For instance, Graham and Leary (2011) document empirical evidence that R&D investment affects the cross-sectional variation in corporate leverage.
[2] Without loss of generality, readers can think this firm as a listed firm in the Nasdaq exchange, which is of importance and likely to face the intertwined problems among corporate leverage, intangibility, inventors' wage, tax, innovation, etc. According to the literature (Strebulaev and Yang, 2013; Bessler et al., 2013), zero-leverage firms are concentrated in the information technology and healthcare sectors, which are highly innovative industries many Nasdaq firms belong to. For Nasdaq firms, we emphasize that corporate innovations affect not only leverage, but also many other corporate characteristics such as inventors' wage and leverage-tax sensitivity.
[3] Not every firm engages in corporate innovation. For instance, between 1950 and 2018, about one quarter of U.S. firms have no experience in R&D. We use "R&D benefit (value)" and "innovation benefit (value)" interchangeably, as R&D and innovation are not separable in our model.

leverage-tax sensitivity.[4]

We consider only technological innovation[5] in our baseline model but extend to product innovation[6] later. Our baseline model generates three sets of implications. First, corporate innovation discourages debt use, as debt use delivers financial distress risk that could make firms lose inventive human capital in place as well as R&D benefits attached to human capital. As debt use increases, the expected duration of inventor employment shortens, and the value of available R&D benefits decreases. When choosing the debt level, firms trade off traditional leverage value (tax benefits on debt minus bankruptcy costs) against R&D-value loss. Hence, the target leverage predicted by our model is lower than that by traditional models without innovation. The detrimental effect of debt on available R&D benefits notably increases with R&D efficiency and the reactions of earnings to technology progress. If inventors in place possess high R&D efficiency or earnings react to innovation success strongly, firms will choose almost-zero leverage and invest heavily in R&D. This provides an R&D-based explanation for the almost-zero-leverage anomaly (Strebulaev and Yang, 2013; El Ghoul et al., 2018). Using reasonable R&D efficiency and earnings parameterization, our calibrated model can simultaneously replicate zero leverage and generate high R&D intensity (R&D expenditures scaled by sales) that matches its empirical counterpart in the sample of almost zero-leverage firms.

Second, we reconcile the mixed results[7] in the literature regarding the relationship between firm leverage and labor costs, using the trade-off between the risk compensation (RC) effect from

[4] Empirical studies have a debate over the cross-sectional relation between corporate leverage and employees' wages or compensation. This relation can be positive (Agrawal and Matsa, 2013; and Chemmanur et al., 2013) or negative (Hanka, 1998; Simintzi et al., 2015; and Michaels et al., 2019). The persistence of the zero-leverage anomalies has been documented by Strebulaev and Yang (2013), El Ghoul et al. (2018), and others. Heider and Ljungqvist (2015) discover that tax rises motivate firms to raise leverage, but tax cuts cause an insignificant corresponding effect. Such findings refute traditional tradeoff theories, which predict that firm leverage reacts more to tax cuts than to tax rises.

[5] Technological innovation means cost savings – firms can produce higher output at the same cost.

[6] Product innovation means a new technology that attracts additional demand, such as Apple's iPads, COVID-19 vaccine and drugs, Tesla's electric cars, etc.

[7] Some empirical works find this relation to be positive (e.g., Agrawal and Matsa, 2013; Chemmanur et al., 2013), while others find this relation negative (e.g., Hanka, 1998; Simintzi et al., 2015).

Berk et al. (2010), and the human capital loss (HCL) effect from Michaels et al. (2019). On the one hand, raising leverage makes inventors bear a higher unemployment risk, resulting in wages as the RC for unemployment (Berk et al., 2010). On the other hand, since the expected duration of inventor employment is shorter (the HCL possibility due to financial distress is higher), raising leverage makes firms extract a smaller R&D benefit from inventors, resulting in lower wages (Michaels et al., 2019). We solve equilibrium wages using the exogenous value-sharing rule that implies a fixed pay (employment contract value)-to-performance (available R&D benefit) ratio. Varying firm characteristics simultaneously affects the outcomes of leverage choice and the trade-off between the RC and HCL effects. This simultaneity shapes the endogenous relationship between equilibrium wages and optimal leverage. This relationship is positive (negative) when marginal wage changes due to the RC (HCL) effect dominates. In this sense, our model nests the ones in Berk et al. (2010) and Michaels et al. (2019) as two special cases. Along with the changes in product demand volatility and the pay-to-performance ratio (R&D efficiency and the product demand growth rate), the relationship is positive (negative).

Third, the marginal R&D-value loss due to debt use is concave in the debt level, and its concavity alters the shape of the leverage-to-tax sensitivity. When tax rate changes lead to the imbalance between marginal R&D-value loss at the current debt level and corresponding marginal leverage value, firms will offset the tax-rise (tax-cut) imbalance by raising (lowering) debt use in a relatively larger (smaller) size, to regain optimal capital structure. Such asymmetries in leverage adjustments are attributed to the concavity that makes the sensitivity of marginal R&D- value loss to debt cuts higher than that to debt rises. As a result, optimal leverage reacts more to tax rises than to tax cuts, which is in contrast to most studies but consistent with Heider and Ljungqvist (2015) who document that tax rises motivate firms to raise leverage but tax cuts lead to a little corresponding effect. We also find that available R&D benefit and leverage jointly display asymmetric sensitivities to tax rate changes, because R&D value monotonically decreases with debt level. The negative reaction of available R&D benefits to tax rises is stronger than the corresponding positive reaction to tax cuts. Our results capture empirical regularities in Mukherjee et al. (2017), which find that tax rises significantly impede innovation but tax cuts weakly boost innovation. The effect of taxes on R&D decisions is largely ignored in existing theories.

We extend the baseline model by adding innovation strategy switches and inventor mobility, obtaining novel empirical predictions. We show that innovation strategy switches twist the influence of product price competition on optimal leverage. In response to a rise in price competition, technology-innovation (product-innovation) firms reduce (raise) leverage. In mixed-innovation firms, the leverage-competition relationship has an inverted-U shape.[8]

Firms reduce leverage when inventors in place have a high mobility intensity. Hedging inventor-mobility-based human capital risk increases leverage and reduces R&D benefit.

We contribute to several strands of the literature. First, we are the first to model firm innovation into capital structure trade-off theories (e.g., Leland, 1994; Ju et al., 2005; Hackbarth et al., 2006; and Chen, 2010). Our model simultaneously captures several puzzling leverage features in the cross-section; i.e., almost-zero-leverage anomaly, mixed leverage-wage relationship; and asymmetric leverage-tax sensitivity. We also provide new and empirically testable predictions among corporate leverage, inventor mobility, human capital risk management, and the coordination between innovation strategies and product pricing.

Moreover, we contribute to the small but growing strand of the literature on the implication of human capital mobility for corporate finance and innovation. Lustig et al. (2011) examine the effect of technical capital losses due to employee mobility on executive compensation. Donangelo (2014) analyzes the cross-sectional relationship between stock returns and labor

[8] Mixed innovation means that firms do technology and product innovation simultaneously.

mobility. Liu et al. (2017) study the consequences of inventor mobility for firms' innovation outputs. Israelsen and Yonker (2017) estimate the loss in firm value attributed to key-employee departures. We complement this literature by studying how inventor mobility and related risk management influence corporate debt policies through innovation valuation.

Finally, our paper makes a methodological contribution to the literature on R&D/ innovation theories and related applications (e.g., Klette and Kortum, 2004; Lentz and Mortensen, 2008; Lin, 2012; Malamud and Zucchi, 2019) by proposing a unified theoretical and tractable model to simultaneously capture several puzzling leverage features in the cross-section. Our model has several advantages over existing models: (i) it endogenizes the detrimental influence of debt use on corporate innovation value. This is the key that enables the model to bridge the gap between innovation and capital structure theories. (ii) it captures the empirical implication of human capital market frictions (inventor mobility) as well as the asymmetric effect of corporate tax changes on R&D decisions. (iii) it allows for product innovation and technology innovation simultaneously, and permits the separation between these two types of innovation effects. (iv) it provides an explicit formula for the expected value of R&D/innovation in a parsimonious and analytically tractable way. A detailed comparison among various innovation models is presented in online Appendix A.

The remainder of this paper unfolds as follows. Section 2 presents our model setting and Section 3 solves our model. Section 4 present our numerical analysis. Section 5 and Section 6 extend our baseline model by adding innovation strategy switches and inventor mobility, respectively. Section 7 concludes.

2. Model Setting

Consider a continuous-trading economy supported by a complete filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q})$ satisfying the usual conditions. Agents are risk-neutral, and discount cash flows at the risk-free rate of r. Time varies over $[0,\infty)$, and the representative firm is unlevered at the initial time point 0. It intends to adjust the capital structure by selling a perpetual debt at its par value *D*, and meanwhile, hires a group of inventors to undertake R&D on technology innovation. The financial market is frictionless,[9] but the labor market suffers job-switching frictions.[10] These frictions are quantified using the duration of temporary unemployment $Du > 0$.

A. Production function

The representative firm in our model engages in producing outputs. The production function, $F:\mathbb{R}_+ \to \mathbb{R}_+$, follows Miao (2005) and takes the form of $F(k_t) \equiv k_t^{\gamma}$, where $\gamma \in (0,1)$ denotes the return-to-scale parameter and k_t denotes the input at time *t* with a fixed unit cost $\delta > 0$ set by the firm's supplier.

B. Productivity technology innovation driven by R&D investment

The firm does R&D that creates new technology. If new technology arrives, the firm promptly applies it to production and gives up old technology. Pursuing new technology induces productivity fluctuations. Like the continuous-time patent value model of Weeds (2002),[11] we specify such a technology innovation effect by using the productivity (technology) dynamics

$$dA_t = \mu_A(RD)A_t\,dt + \sigma_A(RD)A_t\,dW_t,^{12} \quad A_0 \equiv \boldsymbol{A} \quad . \tag{1}$$

[9] Financial frictions (e.g., refinancing costs) discourage using leverage but do not directly affect the marginal effect of debt use on available R&D benefit. After taking financial frictions into account, our comparative statics results remain unchanged. For brevity, therefore, we adopt the setting of frictionless financial markets.

[10] Berk et al. (2010) stress the relevance of unemployment risk induced by the firm's financial distress in determining market wages. They argue that firms choosing higher leverage must pay labors more to compensate them for bearing a higher unemployment risk that accompanies firm bankruptcy. This is because financial distress would trigger firm liquidation that makes labors in place lose jobs and suffer subsequent wealth losses. These losses might emerge from the processes of seeking new job opportunities (e.g., job searching costs or employment consulting fees), and hence, increase with the duration of unemployment. In view of these facts, we consider a setting of labor markets with job-switching frictions, which allows us to capture the implication of temporary unemployment due to firm bankruptcy for equilibrium wages as well as the influence of unemployment costs on employment contract valuation.

[11] Innovative technology in practice is often converted into patents, and hence, the value of patents intuitively reflects technology advantages or the level of technology. In Weeds (2002), the patent value evolves according to a geometric Brownian motion, which inspires and partially justifies our specification of the technology dynamics (expression (1)).

[12] Physical investment can be an alternative channel through which firms improve productivity. Undertaking physical investment could cause an auxiliary effect on the operation of R&D/innovation, but it does not qualitatively change the influence of debt use on available R&D benefits. Since issues on the intersection of physical and R&D

where μ_A is the expected rate of technology progress driven by R&D, and σ_A is innovation risk that captures the uncertainty about innovation outcomes driven by R&D. Without loss of generality, we specify μ_A and σ_A using two linear functions of R&D investment: $\mu_A(RD) \equiv \Lambda_\mu RD$ and $\sigma_A(RD) \equiv \Lambda_\sigma RD$.[13] The ratio of the marginal technology progress rate to marginal technology innovation risk can be employed as a natural measure of R&D efficiency— $\Lambda_\mu / \Lambda_\sigma$.[14] W is a Brownian motion. *RD* denotes the amount invested in R&D.

Our specifications of the technology progress rate and innovation uncertainty are inspired by Lin (2012) and Caggese (2012), respectively. Without allowing for uncertainty on the outcomes of innovation, Lin (2012) assumes that R&D investment can be perfectly converted into intangible capital that delivers endogenous technology progress. Caggese (2012) specifies the effect of innovation success (failure) on technology using technology advantages (disadvantages). In his model, innovation un- certainty has no relationship with R&D. Our specifications are more generalized than these two works. Following Li (2011) and Gu (2016), we assume the amount of R&D investment to be stationary. Pennetier et al. (2019) offer evidence that firms with a persistent R&D spending allocation policy (allocations remain constant) achieve better R&D performance than firms with a dynamic R&D spending allocation policy. Firms pursuing the optimization of R&D performance would prefer the strategy that maintains a fixed R&D spending level to the strategy that adjusts R&D spending frequently. Such empirical implications rationalize our setting of stationary R&D investment.

C. Product demand

investment are beyond the scope of our study, we focus on R&D investment when specifying the productivity dynamics.

[13] We can easily extend the model by using the generalized nonlinear specifications of innovation risk and technology progress, e.g., $\mu_A(RD) \equiv RD^{\Lambda_\mu}$ and $\sigma_A(RD) \equiv RD^{\Lambda_\sigma}$. Such nonlinear specifications do not qualitatively alter our main results.

[14] In our model, the partial derivative of the expected sales growth rate with respect to R&D investment, equivalent to R&D ability defined by Cohen et al. (2013), strictly increases (decreases) with Λ_μ (Λ_σ).

We follow Pichler et al. (2008) and assume the firm's aggregate product demand as

$$\hat{Q}_D(p_t, Q_t) \equiv q(p_t)\,Q_t \equiv p_t^{-\varepsilon} Q_t$$

where p_t is the product price at time t, $q(\cdot)$ is the size of the customer base, ε denotes the customer-to-price elasticity, and Q_t is the average demand per customer. This demand evolves according to $dQ_t = \mu_q Q_t\,dt + \sigma_q Q_t\,dB_t$, $Q_0 \equiv \boldsymbol{Q}$. μ_q is the growth rate and σ_q is the volatility rate. Product demand is unaffected by the outcomes of technology innovation (because these outcomes do not alter product functionality or features), so that the two Brownian motions, B and W, are mutually independent.

D. Operating earnings and assets' aggregate value

Instantaneous before-tax operating earnings at time *t* are given by:

$$\hat{\pi}(p_t, Q_t, A_t) = \underbrace{p_t\,\hat{Q}_D(p_t, Q_t)}_{sales\ revenue} - \underbrace{\delta k^*(p_t, Q_t, A_t)}_{production\ costs} \tag{2}$$

where $k^*(p_t, Q_t, A_t) = p_t^{-\varepsilon/\gamma} Q_t^{1/\gamma} A_t^{-1/\gamma} = k_t^*$ is the equilibrium demand for inputs and is derived from the market-cleaning condition $\hat{Q}_D(p_t, Q_t) = \hat{Q}_S(A_t, k_t^*) \equiv A_t F(k_t^*)$. Hence, the aggregate value of firm assets takes a standard form of the total present value of future earnings flows:

$$V_t(RD) = \mathrm{E}_t \int_t^{\infty} \hat{\pi}(p_s, Q_s, A_s)\,\mathrm{e}^{-r(s-t)}\,ds\,.$$

where E_t is the expectation conditional on F_t. Note that as in Leland (1994), payout decisions are not considered in our model. Any net cash outflow associated with R&D expenditures, wage payments, and debt service will be financed through selling additional equity.[15,16]

[15] Retained earnings can be an alternative source of funding for R&D spending. Making the choice of financing source between retained earnings and new stock issuance is meaningless to the firm, since the stock market is assumed to be frictionless. Moreover, the inclusion of retained earnings does not create additional insights about our current results.

E. Separation between tangible asset value and intangible asset value

Tangible assets include equipment, property, or plant, while intangible assets refer to goodwill, knowledge capital, or technology advantages attached to employees' human capital, etc. In the model, we define intangible assets as technological advantages created by innovation success. To separate intangible asset value from aggregate asset value, we consider an industrial technology benchmark $\boldsymbol{A} = A_0$, which helps us measure how large the advantages of technological progress in asset valuation are.

Given the industrial technology benchmark, we calculate tangible asset value by using

$$V_{T,t} = \mathrm{E}_t \int_t^{\infty} \hat{\pi}(p_{T,s}, Q_s, \boldsymbol{A}) \mathrm{e}^{-r(s-t)} ds$$

where $p_{T,\cdot}$ is the product price in the absence of innovations. We use $p_{T,\cdot}$ only when specifying earnings in the case of no innovation and solving tangible asset value. Note that $p_{T,\cdot}$ should be distinguished from the product price under innovation *p*. In the absence of innovation, technology affects the initial product price, but does not affect subsequent product price adjustments. In the presence of innovation, the product-price dynamics are partially driven by technology fluctuations. Since the valuation of tangible assets should not include the effect of innovation, technology is fixed at its industrial benchmark level. The value of intangible assets simply equals to aggregate asset value minus the value of tangible assets, i.e., $V_{I,t}(RD) = V_t(RD) - V_{T,t}$ for any time $t \geq 0$.

F. Employment contract

Hence, to restrict attention to the role of debt use in assessing innovation value, our model does not consider retained earnings.

[16] Several empirical papers have documented that equity is preferable to debt in financing innovative projects. For example, Stigliz (1985) finds that the uncertainty and volatile return of innovative projects make them unattractive to creditors. Hall and Lerner (2010) mention that intangible assets created by innovation are difficult to quantify as collateral for debt financing. Equity capital is a favorable way to finance innovation because it allows equity holders to share upside returns and does not require collateral.

We follow Berk et al. (2010) and assume managers hire a group of inventors by committing to a long-term contract with a static wage setting, which pays wages I continuously.[17] The wage level is determined according to a fixed pay-performance ratio $\beta \in [0,1]$ that embodies the value-sharing rule between shareholders and inventors.[18] In the specification of this ratio, *pay* refers to the net value of the employment contract, while *performance* refers to the value of innovation benefits contributed by inventors. The contract requires inventors to abandon all outside job options. This implies the assumption of no labor mobility, which will be released in Section 6. Managers unilaterally abrogate the employment contract if and only if the firm goes bankrupt, and do not make severance payments after bankruptcy (as in Berk et al., 2010).[19] The occurrence of bankruptcy forces inventors in place to seek new jobs and to suffer subsequent wealth losses arising from job-switching frictions. During temporary unemployment, inventors continuously spend φI on job-seeking, where φ is the job-seeking cost rate. Job-seeking spending, which includes job searching cost, the charge for labor intermediate services, and employment consulting fees, plays a role as the costs of unemployment embedded into the employment contract.

G. Debt contract

Now we consider a standard non-callable perpetual debt contract. The debt continuously

[17] Section 6 extends our baseline model by allowing for dynamic wage adjustments, which examines the firm's behavior in hedging against human capital risk due to inventor mobility.

[18] The concept of the value-sharing rule is similar to the surplus-sharing rule in Michaels et al. (2019). That article quantifies workers' bargaining power using the ratio of worker surplus to the sum of worker surplus and firm surplus. The pay-performance ratio in our model, hence, can be thought of as an alternative proxy for the bargaining power of inventive labors. The dispersion of the pay-performance ratio is intuitively attributed to the influence of permanent fluctuations in labor demand and supply on labor market competition. Issues related to labor market dynamics (e.g., labors' competitive behavior in job seeking, the impact of human capital scarcity on employment decisions, human capital allocation in industries, etc.) are beyond the scope of our research, and hence we consider an exogenous value- sharing rule and assume the pay-performance ratio to be stationary, which exogenizes the influence of labor market competition and bargaining on employment contract design as a given requirement for employment.

[19] The assumption that managers' tolerance toward innovation failure is unlimited can be easily released. Our results remain unchanged in the case of limited innovation failure tolerance. Model extension with taking limited tolerance for innovation failure into account is available in online Appendix B7.

pays interest d that shields operating earnings from taxes at the rate τ. As in Black and Cox (1976), Ju et al. (2005), and the exogenous bankruptcy case of Leland (1994), the debt contract has a covenant specifying that the firm is forced into bankruptcy if its after-tax asset liquidation value falls below the debt principal. According to this covenant, we define the arrival of firm bankruptcy as a random stopping time: $T_d := \inf(t > 0 : (1-\tau) V^*_{T,t} \leq D)$. Debt holders recover a portion $1-\alpha$ of the remaining tangible asset value after firm liquidation. The portion α is consumed by the bankruptcy process and asset fire sales. Debt recovery does not contain intangible assets, because financial distress causes inventor departures, and technology advantages are always attached to inventors' human capital. While few empirical studies find that some intangible assets (e.g., patents) can be pledged as collateral, our assumption on debt-recovery claims does fit most firms[20].

3. Solving the Model and Characterizing Firm Policies

Section 3 solves the model in four steps. The first step derives the optimal product price and further incorporates optimal pricing strategies into the specification of firms' earnings. Given this specification, the second step offers the pricing formula for R&D on technology innovation. The third step solves endogenous wages for inventors hired by innovating firms. The first three stages take debt usage as given. The final step characterizes firms' debt policy by taking R&D investment optimization into account. Technical proofs for model solutions are shown in online Appendix B. We construct Figure 1 to facilitate understanding the structure of our model.

[20] For instance, from 1990 to 2013, the annualized fraction of U.S. patenting firms that pledge their patents as collateral roughly ranges from 10% to 38% (see Mann, 2018). The average proportion of firms with the pledgeability of patents to whole innovating firms is probably smaller than 15%. Besides, only about 3%-15% of patents have been pledged within five years of being granted. As the fraction of innovating firms with patent collateral is low, our model does not consider debt-recovery claims on intangible assets.

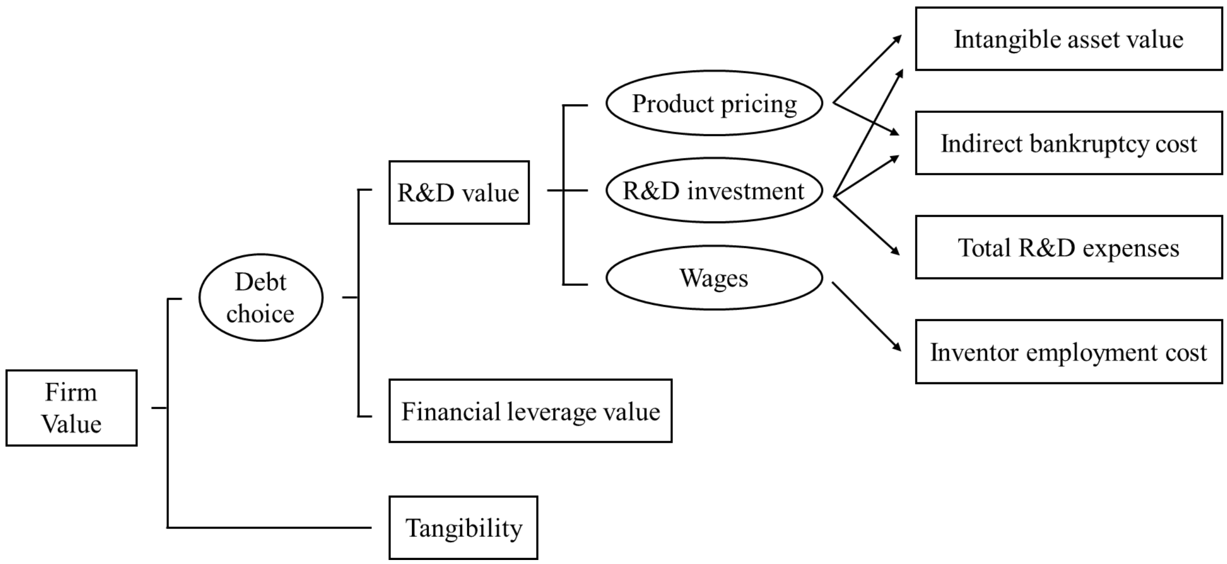

Figure 1: The unified structure of firm value and firm decisions.

A. Product pricing strategy

We derive the optimal product price from the objective of maximizing instantaneous earnings. Differentiating the earnings function (2) with respect to p_t, setting this expression being zero with $p_t = p_t^*$, and solving for p_t^*, we obtain the following proposition.

Proposition 1. (Optimal Product Pricing and Asset Value) *Given the earnings function as* (2), *the firm sells its products at* $p_t^* = Q_t^{(1-\gamma)/\eta} A_t^{-1/\eta} (\frac{\delta\varepsilon}{\varepsilon-\eta})^{\gamma/\eta}$. *Initial aggregate asset value under optimal product pricing takes the following form:*

$$V_0^*(RD) = \mathrm{E}_0 \int_0^\infty \hat{\pi}^*(Q_t, A_t) \mathrm{e}^{-rt} ds$$

where $\hat{\pi}^*(Q_t, A_t) = A(Q_t, A_t)\Phi$ *denotes instantaneous earnings under optimal product pricing,* $A(a,b) \equiv a^{1/\eta} b^{(\varepsilon-1)/\eta}$, $\Phi \equiv \left(\frac{\delta\varepsilon}{\varepsilon-\eta}\right)^{1-\varepsilon/\eta} \frac{\eta}{\varepsilon}$, *and* $\eta \equiv \varepsilon + \gamma - \varepsilon\gamma$. *The optimal product price strictly increases with product demand* ($\partial p_t^* / \partial Q_t > 0$) *but decreases with technology* ($\partial p_t^* / \partial A_t < 0$).

In the model, product pricing plays a twofold role. Firstly, it is a key channel through which firms absorb innovation benefits. Firms can convert intangible technology progress arising from successful innovation into a tangible increase in operating earnings by lowering the product price (note $\partial p_t^* \big/ \partial A_t < 0$ and $\partial \hat{\pi}^* \big/ \partial A_t > 0$).

Secondly, it helps clarify how changes in product price competition influence the firm's leverage decisions through innovation incentives. According to Proposition 1, the reactions of earnings to technology progress increase with price elasticity, since $\partial((\varepsilon - 1)\eta^{-1}) \big/ \partial \varepsilon > 0$. Tougher price competition enables firms to earn larger benefits from innovation, and thus encourage innovation. Such an effect could exacerbate the consequence of debt use for available innovation benefits, and further lower firms' incentives to use leverage. Section 5B presents detailed discussions on the interaction among firm leverage, innovation, and price competition.

The form of earnings under optimal product pricing in the absence of innovation $\hat{\pi}^*(Q_t, \boldsymbol{A})$ can be derived similarly. For notational consistency, we let $V_{T,t}^*$ denote tangible asset value under optimal product pricing.

B. R&D investment in innovation valuation

Now we consider the valuation of innovation (or R&D). Undoubtedly, the decision problem on R&D investment is at the heart of innovation valuation. We formulate this decision problem under optimal product pricing, which can be given by

$$\max_{RD \geq 0} V^{Performance}(\boldsymbol{A}, \boldsymbol{Q}; D, RD) \tag{3}$$

where

$$V^{Performance}(\boldsymbol{A}\ ,\boldsymbol{Q};D,RD)\equiv \underbrace{\mathrm{E}_{0_+}\int_{0_+}^{T_d}[\hat{\pi}^*(Q_t,A_t)-\hat{\pi}^*(Q_t,\boldsymbol{A})]\,\mathrm{e}^{-rt}\,dt}_{the\ output-side\ effect\ on\ CFs}\ \underbrace{-\mathrm{E}_{0_+}\int_{0_+}^{T_d}RD\,\mathrm{e}^{-rt}\,dt}_{the\ input-side\ effect\ on\ CFs}$$

denotes the value of available R&D benefit contributed by inventors. Its measurement consists of the output-side effect and the input-side effect on cash flows. We assess the output-side effect by computing the marginal effect of the use of new technology (R&D output) on earnings. This effect is positive $\hat{\pi}^*(Q_t,A_t)-\hat{\pi}^*(Q_t,\boldsymbol{A})>0$ when new technology delivers a good outcome that raises productivity or enhances technology advantages. The effect is negative $\hat{\pi}^*(Q_t,A_t)-\hat{\pi}^*(Q_t,\boldsymbol{A})<0$ if the outcome of innovation is bad and it destroys technology advantages ($A_t<\boldsymbol{A}$). The input-side effect is quantified using R&D investment RD (R&D input).

To better understand the implications behind R&D value, we decompose its expression into three parts: (a) intangible asset value V_I^*; (b) indirect bankruptcy costs due to inventor departures V^{IBC}; and (c) total expected R&D expenses $V^{Expense}$. Their expressions are given by

$$V^{Performance}=V_I^*-V^{IBC}-V^{Expense},$$

$$V_I^*\equiv \mathrm{E}_{0_+}\int_{0_+}^{\infty}\hat{\Pi}(Q_t,A_t)\,\mathrm{e}^{-rt}\,dt,\ \ V^{IBC}\equiv \mathrm{E}_{0_+}\mathrm{e}^{-rT_d}\,V_{I,T_d}^*,\ \ V^{Expense}\equiv \mathrm{E}_{0_+}\int_{0_+}^{T_d}RD\,\mathrm{e}^{-rt}\,dt,$$

where $\hat{\Pi}(Q_t,A_t)\equiv\hat{\pi}^*(Q_t,A_t)-\hat{\pi}^*(Q_t,\boldsymbol{A})$. The first two terms form R&D benefits, equaling intangible asset value minus indirect bankruptcy costs. The last term captures R&D costs.

Intangible asset value has its own explicit solution:

$$V_I^*(\boldsymbol{A}\ ,\boldsymbol{Q};RD)=A(\boldsymbol{Q},\boldsymbol{A}\ )K(RD)-A(\boldsymbol{Q},\boldsymbol{A})K(0) \tag{4}$$

where the function $K(a)\equiv\Phi\left[r-\frac{(\varepsilon-1)\,a\,\Lambda_\mu}{\eta}-\frac{0.5(\varepsilon-1)}{\eta}\times\frac{(\varepsilon-1-\eta)\,a^2\,\Lambda_\sigma^2}{\eta}-\frac{\mu_q}{\eta}-\frac{0.5(1-\eta)\sigma_q^2}{\eta^2}\right]^{-1}$. On the right-hand side of (4), the first term represents the aggregate value of firm assets under innovation. The second term shows tangible asset value, unaffected by innovation. Intangible asset value can be thought of as expected R&D benefits without allowing for the consequences of debt use.

In contrast, the role of indirect bankruptcy costs is to capture the influence of debt financing on R&D benefits. Specifically, these costs denote intangible-asset loss due to inventor departures accompanying financial distress. Debt use delivers financial distress risk that could cause firms to lose inventive human capital in place as well as R&D benefits (e.g., intangible assets formed by technology advantages) attached to human capital. Briefly speaking, the occurrence of financial distress forces inventors to leave and to take away remaining intangible assets. When assessing expected R&D benefits enjoyed by a firm with debt use, hence, indirect bankruptcy costs should be deducted from intangible asset value.

Indirect bankruptcy costs are measured using the present value of intangible asset loss at the bankruptcy point. These costs resemble a financial security that pays no interest, but has the value equal to intangible asset value at default, i.e., $V^{IBC}(A_{T_d}, Q_{T_d}; D, RD) \rightarrow V^*_{I,T_d}$. Their formula is

$$V^{IBC}(\boldsymbol{A}, \boldsymbol{Q}; D, RD) = J(\boldsymbol{A}, \boldsymbol{Q}_d(D)) \int_{0_+}^{\infty} H(t)\, f_{0_+}^{T_d}(t; D)\, dt - V^*_{T,T_d}\, Q\,(\boldsymbol{Q}, \boldsymbol{Q}_d(D), x, y) \quad (5)$$

where $J(a,b) \equiv A(b,a)K(RD)$, $Q(a,b,c,d) \equiv (ab^{-1})^{-c-d}$, $H(t) \equiv \mathrm{e}^{\{[\Lambda_\mu RD - 0.5\Lambda_\sigma^2 RD^2(1-\upsilon)]\upsilon - r\}t}$, $\boldsymbol{Q}_d(D) \equiv Q_{T_d} = D^{\eta}[(1-\tau)K(0)]^{-\eta}\boldsymbol{A}^{1-\varepsilon}$, the constants $\langle \upsilon, x, y \rangle$ and the default density function $f_{0_+}^{T_d}(\cdot)$ are all defined in online Appendix B2.

We next discuss the implication of R&D costs. In the model, RD is equivalent to the firm's instantaneous R&D expenses. Total expected R&D expenses, hence, can be calculated as the total present value of instantaneous R&D expenses in the future. Their formula is given by

$$V^{Expense}(\boldsymbol{Q}; D, RD) = RD\, r^{-1}\left[1 - Q\,(\boldsymbol{Q}, \boldsymbol{Q}_d(D), x, y)\right]. \quad (6)$$

The term outside (inside) the square brackets captures the capitalized value of total R&D expenses (the firm's survival probability conditional on initial product demand). At the bankruptcy point, the value of total expected R&D expenses approaches zero, i.e., $V^{Expense}(\boldsymbol{Q}_d(D);D,RD)\to 0$. This means that the occurrence of bankruptcy gives rise to inventor departures, and hence, forces managers to suspend R&D investment.

We describe the decision rule of optimal R&D investment in the following proposition.

Proposition 2. (Optimal R&D Investment) *Given an arbitrary debt choice D and the objective function* (3)*, optimal R&D investment* $RD^*(D)$ *satisfies the first-order condition* (*FOC*):

$$J\,(\boldsymbol{A}\;,\boldsymbol{Q})-\int_{0_+}^{\infty}[\,J\,(\boldsymbol{A}\;,\boldsymbol{Q}_d(D))+J\,(\boldsymbol{A}\;,\boldsymbol{Q}_d(D))H\,(t)]\,H\,(t)\,f_{0_+}^{T_d}(t;D)\,dt$$
$$=r^{-1}[1-Q\,(\boldsymbol{Q},\boldsymbol{Q}_d(D),x,y)],$$

where the functions $J\,(u,v)\equiv A\,(v,u)K\,(RD^*(D))$, $H\,(t)\equiv t\,\upsilon\,[\Lambda_\mu-RD^*(\,D\,)\,\Lambda_\sigma^2\,(1-\upsilon)]$, *and* $K\,(a)\equiv[K(a)]^2\,\Phi^{-1}t^{-1}H\,(t)$. *The left-hand side of the FOC represents the marginal expected R&D benefit. The right-hand side of the FOC represents the marginal expected R&D cost.*

Debt use changes respectively affect marginal R&D benefit and cost via default density $f_{0_+}^{T_d}(\cdot)$ and the default threshold $\boldsymbol{Q}_d(D)$. Raising debt use shortens the expected duration of inventor employment, increases default likelihood, and makes the default threshold higher. As a result, firms earn a smaller R&D benefit from inventors, the cumulative amounts of R&D funding spent by inventors are fewer, and both marginal R&D benefit and cost become lower. Raising debt use will cause firms to reduce (expand) R&D investment if the debt-induced decrement in marginal R&D benefit (cost) outweighs that in marginal R&D cost (benefit). We find the influence of debt use on R&D investment under most circumstances to be robustly negative.

C. Endogenous wages

Now we derive the level of wages under partial equilibrium I^*. Since inventors are risk-neutral, the net present value of their employment contract is

$$V^{Pay}(\boldsymbol{Q};D) = V^{Wage}(\boldsymbol{Q};D) - V^{JSC}(\boldsymbol{Q};D) \equiv \mathrm{E}_{0_+}\int_{0_+}^{T_d} I\,\mathrm{e}^{-rt}dt - \mathrm{E}_{0_+}\mathrm{e}^{-rT_d}\int_{T_d}^{T_d+Du} \varphi\, I\,\mathrm{e}^{-r(t-T_d)}dt \tag{7}$$

where the first term on the right-hand side captures the total present value of wage flows. The second term captures the present value of aggregate job-seeking costs throughout temporary unemployment triggered by firm bankruptcy. These costs, equivalent to inventors' wealth losses arising from job-switching frictions, constitute the unemployment costs embedded into their employment contract.

Section 2F demonstrates that inventors and shareholders share available R&D benefits following the value-sharing rule that implies a fixed pay-performance ratio. Endogenous wages, hence, can be solved from $\beta V^{Performance} = V^{Pay}$.

Proposition 3. (Endogenous Wages) *Given the formula for available R&D benefits* (*expressions* (4)-(6)), *the employment contract* (7), *the pay-performance ratio* β, *and an arbitrary debt choice D, endogenous wages under optimal R&D policy take an explicit form:*

$$I^* \equiv I^*(D;\boldsymbol{A},\boldsymbol{Q},\beta) = \frac{\beta V^{Performance}(\boldsymbol{A},\boldsymbol{Q};D,RD^*(D))}{V^{Multi}(\boldsymbol{Q};D)}$$

where $V^{Multi}(\cdot) \equiv r^{-1}[1+(\varphi\,\mathrm{e}^{-rDu}-\varphi-1)Q(\boldsymbol{Q},\boldsymbol{Q}_d(D),x,y)]$ *is the value multiplier for pricing employment contracts. This value multiplier satisfies* $V^{Pay} = I^* V^{Multi}$. *The influence of changes in debt use on endogenous wages consists of two parts:*

$$\frac{\partial I^*}{\partial D} = \underbrace{\frac{\partial I^*}{\partial V^{Performance}} \times \frac{\partial V^{Performance}}{\partial D}}_{human\ capital\ loss\ effect} + \underbrace{\frac{\partial I^*}{\partial V^{Multi}} \times \frac{\partial V^{Multi}}{\partial \boldsymbol{Q}_d(D)} \times \frac{\partial \boldsymbol{Q}_d(D)}{\partial D}}_{risk\ compensation\ effect}$$

where the risk compensation (*RC*) *effect is positive* (*because* $\frac{\partial V^{Multi}}{\partial \boldsymbol{Q}_d(D)} < 0$ *and* $\frac{\partial \boldsymbol{Q}_d(D)}{\partial D} > 0$) *and the human capital loss* (*HCL*) *effect is negative* (*because* $\frac{\partial V^{Performance}}{\partial D} < 0$).

In the model changing debt use affects endogenous wages through two channels, including available R&D benefit and employment contract value. On the one hand, increasing debt use will make firms derive a smaller R&D benefit from inventors $\frac{\partial V^{Performance}}{\partial D} < 0$. This is because as debt use rises, the likelihood of HCL due to financial distress increases, and the expected duration of inventor employment shortens. Such a negative effect on available R&D benefits motivates firms to lower wages.

On the other hand, the occurrence of bankruptcy forces inventors to lose their jobs, so raising debt use makes them bear a higher unemployment risk and makes expected unemployment costs higher. The value of the employment contract displays a negative sensitivity to debt use (if holding wages fixed), which means $\frac{\partial V^{Multi}}{\partial \boldsymbol{Q}_d(D)} \times \frac{\partial \boldsymbol{Q}_d(D)}{\partial D} < 0$. Given this fact, firms pay higher wages as the RC for unemployment. The debt-wage relationship depends on the trade-off between the RC and HCL effects. For related further discussions, please see Section 4C.

D. Debt choice

We have been aware that debt use may lower available R&D benefits by increasing the likelihood of human capital losses due to financial distress. When making the debt choice, firms should consider the trade-off between R&D value and traditional leverage value. The objective is to choose an optimal debt level that maximizes the firm's total value equaling the sum of after-tax tangible asset value, after-tax net R&D value, and traditional leverage value:

$$\max_{D \geq 0} \left[(1-\tau) V_{T,0}^{*} + (1-\tau) V^{RD}(\boldsymbol{A}, \boldsymbol{Q}; D, RD^{*}(D)) + V^{FL}(\boldsymbol{Q}; D) \right] \tag{8}$$

where we measure net R&D value using available R&D benefit minus wage-based compensation paid to inventors $V^{RD} \equiv V^{Performance} - V^{Wage}$. V^{FL} represents leverage value, equaling debt's tax benefit minus direct bankruptcy cost due to asset fire sales (as in Leland, 1994):

$$V^{FL}(\boldsymbol{Q}; D) = \underbrace{\tau\, d\, r^{-1}[1 - Q\,(\boldsymbol{Q}, \boldsymbol{Q}_d(D), x, y)]}_{tax-shield\ benefits} - \underbrace{\alpha\, D\, Q\,(\boldsymbol{Q}, \boldsymbol{Q}_d(D), x, y)}_{direct\ bankruptcy\ costs}.$$

The optimal rule of debt choice is presented in the following proposition.

Proposition 4. (Optimal Debt Choice) *The trade-off between R&D value and traditional leverage value reaches equilibrium if and only if their marginal rates of substitution are equal. This trade-off equilibrium determines the optimal debt choice* D^{*}*, which satisfies*

$$\frac{\partial V^{FL}(\boldsymbol{Q}; D^{*})}{\partial D^{*}} = -\frac{\partial \left[(1-\tau) V^{RD}(\boldsymbol{A}, \boldsymbol{Q}; D^{*}, RD^{*}(D^{*})) \right]}{\partial D^{*}}. \tag{9}$$

The right-hand side of equation (9) *represents marginal R&D-value loss due to debt use, while the left side represents marginal traditional leverage value.*

Proposition 4 can be viewed as a modified version of the capital structure trade-off theory that allows for R&D investment in innovation. Marginal R&D-value loss, equivalent to an additional cost of debt use, elucidates the consequence of debt use for available R&D benefit. Therefore, firms pursuing R&D benefits may be discouraged to use debt. The right-hand side of equation (9) being zero generates the optimal rule of debt choice without considering innovation, i.e., $\partial V^{FL}(\boldsymbol{Q};D^{NI})/\partial D^{NI}=0$. Marginal leverage value at optimization in the absence of innovation equals zero, but that in the presence of innovation is positive $\partial V^{FL}(\boldsymbol{Q};D^{*})/\partial D^{*}>0$. Such dramatic differences deliver an inference —debt usage chosen by innovating firms is lower than that by non-innovating firms $D^{*}<D^{NI}$. This is because leverage value has an inverted-U relationship with debt use, and marginal leverage value decreases with debt use. This inference will be verified numerically in Section 4B.

4. Numerical Analysis

This section conducts numerical analysis to study the quantitative implications of technology innovation for leverage features in the cross-section. We use the case of no innovation (choosing $\Lambda_{\sigma}=\Lambda_{\mu}=0$) as our benchmark model for comparison.

A. Parameter calibration

We calibrate baseline parameters at the values that roughly reflect a typical U.S. corporation (the baseline parameter values are summarized in Table 1). We choose the initial average product demand per customer $\boldsymbol{Q}$ at 100, the input unit cost δ at \$1, industrial technology benchmark $\boldsymbol{A}$ at 1, initial technology $\boldsymbol{A}$ at 1, and the job-seeking cost rate at 1. Although these parameter choices can be motivated in more details, we omit these for brevity. Our major results only vary quantitatively but not qualitatively with them. Following Chu (2012), we set the return-to-scale parameter γ at 0.6. We choose the price elasticity of the customer base ε at 1.25, which is in the range of empirical estimates from Hoch et al. (1995) and Foster et al. (2008). We set the duration of temporary unemployment Du to be 23 weeks. The 2019 annual report of the U.S. Bureau of Labor Statistics shows that the average duration of unemployment is 22.1 weeks. Similarly, the

Table 1: Parameter Definition and Baseline Values.

Parameter Value	Parameter Definition
$\boldsymbol{Q} = 100$	Initial average product demand per customer
$\mu_q = 1.6\%$	Product demand growth rate
$\sigma_q = 26.4\%$	Product demand volatility rate
$\boldsymbol{A} = 1$	Industrial technology (productivity) benchmark
$\boldsymbol{A} \;\; = 1$	Initial technology (productivity) level
$\Lambda_\mu = 5\%$	Marginal technology progress rate
$\Lambda_\sigma = 16.5\%$	Marginal technology innovation risk
$\varepsilon = 1.25$	Price elasticity of the customer base
$\gamma = 0.6$	Return-to-scale parameter
$\delta = \$1$	Input unit price
$r = 6\%$	Riskless interest rate
$Du = 23\,\text{weeks}$	Duration of temporary unemployment
$\varphi = 1$	Job-seeking cost rate
$\tau = 32\%$	Corporate tax rate
$\alpha = 25\%$	Bankruptcy cost rate
$\beta = 5\%$	Pay-performance ratio

survey commissioned by Randstad in 2018 finds that U.S. job applicants take an average of five months to get a new job.

Next, we calibrate the parameters governing firm innovation and product demand. We choose the marginal technology progress rate Λ_μ at 5%, marginal innovation risk Λ_σ at 16.5%, product demand risk σ_q at 26.4%, and the product demand growth rate μ_q at 1.6%, respectively. Given these four parameter choices, our model at optimization generates several financial variables that roughly match the following numbers. First, the cash flow growth rate is equal to 2.5%, consistent with the target value used by Miao (2005). Second, cash flow volatility is moderately targeted at 25.5%. Strebulaev (2007) finds the estimated volatility of firm-level cash flows to be 25.5%. Third, the ratio of optimal R&D expenditure to total sales (R&D intensity) is 4.64%. This value roughly matches empirical estimates for the mean R&D intensity of U.S. firms

(Cazier, 2011; Kusnadi and Wei, 2017). Fourth, the annualized elasticity of productivity to innovation (proxied by cumulative R&D investment) equals 0.114, which is in the range estimated by Kogan et al. (2017).[21, 22]

We set the fixed pay-performance ratio β at 5%. In the model, this parameter can be thought of as the fixed pay-performance sensitivity (PPS). Our choice is close to the calibration results of PPS in He (2011), about 5.33%. Following Leland (1994), we set the riskless interest rate r at 6%. The corporate tax rate τ is chosen at 32%, as the estimated marginal tax rate roughly lies in a range from 31.4% to 32% (see Carlson and Lazrak, 2010; Graham and Leary, 2011). We set the bankruptcy cost parameter α at 25%. While this number is higher than the results in Andrade and Kaplan (1998) that report the distress costs of 10%-20% (as a fraction of firm value), it is in the range of recent empirical estimates. Korteweg (2010) finds the estimated distress cost to be 15%-30% of asset value. Davydenko et al. (2012) estimate that distress costs in large firms are sizeable, at least 20%-30% of asset market value.

B. Influence of firm innovation on optimal leverage

Now we consider the influence of innovation on optimal leverage, which is measured as the ratio of debt value to firm value D^*/FV where $FV = (1-\tau)V^*_{T,0} + (1-\tau)V^{RD} + V^{FL}$. Major model outputs are reported in Table 2.

According to Table 2, the firm's desire for R&D benefit heavily discourages using debt. In the absence of innovation, the model delivers a 33.84% leverage ratio accompanied

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[21] The annualized elasticity of expected productivity to innovation ξ is computed using

$$\xi = (\partial EP/\partial Inno)EP^{-1}T^{-1}Inno,\ \ Inno \equiv \mathrm{E}_{0_+}\int_{0_+}^{T\wedge T_d} RD^*(D^*)\,\mathrm{e}^{-rt}\,dt,\ \ EP \equiv \mathrm{E}_0 A_T\cdot$$

This elasticity is independent of the choice of T, since R&D investment, the drift rate, and the volatility of productivity are all time independent.

[22] Kogan et al. (2017) estimate that, on average, a one-standard deviation increase (around 12.45%) in innovation is associated with a 0.6% to 3.5% increment in total factor productivity (TFP), depending on the specifications. These results imply a range of the TFP-innovation elasticities from 0.0468 to 0.2734.

Table 2: Simultaneous Optimization of Debt and R&D Policies.

This table gives various model outputs at the simultaneous optimization of R&D investment and capital structure. In the upper panel the columns (from left to right) present market leverage, debt choice, equity value, coupon level, tax benefit scaled by firm value, direct bankruptcy cost scaled by firm value, and net leverage value scaled by firm value. In the lower panel, the columns (from left to right) present the ratio of R&D expenditure to total sales, the annualized elasticity of productivity to cumulative R&D investment, total expected after-tax R&D expenses scaled by firm value, after-tax indirect bankruptcy costs scaled by firm value, wages, and net after-tax R&D value scaled by firm value.

Model type	Capital structure outputs						
	Market leverage	Debt choice	Equity value	Coupon level	TB scaled by FV	DBC scaled by FV	Net FL value scaled by FV
Benchmark	33.84%	$219.2	$428.71	$14.56	8.391%	2.538%	5.852%
Basic	14.01%	$116.2	$712.95	$7.258	4.005%	0.498%	3.507%
$\Lambda_\mu = 7\%$	3.06%	$39.2	$1242.71	$2.374	0.949%	0.030%	0.919%
$\Lambda_\sigma = 12\%$	3.77%	$45.9	$1172.31	$2.789	1.163%	0.045%	1.118%
$\varepsilon = 1.5$	1.65%	$13.9	$828.21	$0.836	0.521%	0.007%	0.513%

Model type	R&D/Innovation outputs					
	R&D intensity	Prod.-R&D elasticity	R&D cost scaled by FV	IBC scaled by FV	Wages	Net R&D value scaled by FV
Basic	4.641%	0.114	2.682%	1.661%	$1.033	22.916%
$\Lambda_\mu = 7\%$	6.677%	0.230	2.794%	0.586%	$3.195	51.493%
$\Lambda_\sigma = 12\%$	8.964%	0.221	3.913%	0.738%	$2.904	48.807%
$\varepsilon = 1.5$	10.029%	0.156	4.117%	0.351%	$2.411	60.555%

with 5.852% net tax benefits as a fraction of firm value (FL value in Table 2). Leverage and net tax benefits respectively fall to 14.01% and 3.507% after taking innovation into account. The numbers obtained from our model are close to corresponding empirical counterparts, whereas those from the benchmark model seem counterfactually high. Bena and Garlappi (2020) find that the median market leverage ratios of innovating firms and non-innovating firms in the U.S. equal 16% and 20%, respectively. As for tax benefits on debt, Graham (2000) estimates that the capitalized value of net interest deduction is roughly 4.3% of firm value. Van Binsbergen et al. (2010) find net tax

benefits to be 3.5% of asset value. These suggest that the inclusion of innovation can improve the trade-off model's performance in predicting firm leverage and the value of debt's tax benefits.

In contrast with the benchmark model, leverage ratios from our model are much lower. There are two reasons. The first is that debt use delivers a detrimental effect on available R&D benefits. Financial distress risk due to debt use could make firms lose inventive human capital in place as well as R&D benefits attached to human capital. As debt use increases, the expected duration of inventor employment shortens and indirect bankruptcy costs due to inventor departures become higher. This consequence of deb use makes firms derive a smaller R&D benefit from inventors in place, and therefore, marginally lowers firms' incentives to use debt.

The other reason is that equity and debt holders' rights to share R&D benefits are asymmetric. Earnings increments generated by R&D benefit will be distributed as additional dividends to equity holders. In contrast, debt holders have no right to claim R&D benefits and just receive the fixed interest payment. Besides, R&D performance does not affect tangible asset value as well as bankruptcy risk embedded into debt contract (recall that the timing of bankruptcy only depends on tangible asset value).[23] Hence, undertaking R&D marginally boosts the value of equity, whereas does not make any change in the value of debt (if holding par value fixed). Such an asymmetric value effect further enhances the negative influence of R&D on market leverage.

The above implications offer an alternative explanation for the so-called low-leverage puzzle. Miller (1977) firstly proposes this puzzle that on average firms have low leverage ratios relative to what we may predict from trade-off theories. Our key intuition is that corporate R&D (or innovation) benefits reduce their debt use, and hence, ignoring the effect of innovation causes

[23] Under product innovation, tangible asset value increases with R&D performance. This is because R&D/innovation success delivers product quality improvements that help firms attract new customers. New customers boost earnings through product demand, so that tangible assets appreciate, bankruptcy risk lowers, and leverage use becomes more valuable. These results further imply an enhancement effect of product innovation success on marginal leverage value. Nevertheless, this enhancement effect is not enough to offset corresponding marginal R&D-value loss. Our result that optimal leverage is negatively associated with R&D on innovation remains valid in product-innovation firms. Further discussions on the implication of product innovation for leverage are made in Section 5.

traditional trade-off models to overstate optimal leverage usage. Many other attempts have been made to resolve the low-leverage puzzle by using various economic forces, e.g., agency conflict (Leland, 1998), asset liquidity risk (Morellec, 2001), dynamic restructurings (Goldstein et al., 2001), and business cycle (Chen, 2010). Allowing for these forces, however, existing trade-off models under reasonable parameterizations still fail to capture extreme debt conservatism in firms with zero or almost zero leverage. This failure restricts the performance of existing trade-off models in fitting the cross-sectional distribution of firm leverage, especially the performance in fitting the left tail of leverage distribution.

Strebulaev and Yang (2013) document the relevance of zero-leverage firms and almost zero-leverage firms in shaping the low-leverage puzzle. They find that in the past five decades, around 10% of U.S. firms are zero-levered and 23% of U.S. firms are almost zero-levered (book leverage lower than 5%). To better resolve the almost-zero-leverage puzzle and better improve the trade-off model's ability to explain leverage variations in the cross-section, researchers should explain why some firms use little debt or have zero debt instead of why firms on average are less leveraged than expected. Motivated by these facts, we next examine whether and how the influence of innovation on leverage decisions reconciles the trade-off theory with the almost-zero-leverage anomaly.

A stylized fact documented by Strebulaev and Yang (2013) is that the prevalence of zero leverage has a positive association with corporate R&D. Besides, in their sample, the mean of R&D intensities chosen by zero-leverage firms and almost-zero-leverage firms exceeds 20%, far higher than our calibration target (4.64%). To make parameter calibration better reflect reality, we reset the values of the parameters involving R&D efficiency and price competition before proceeding. We reset the price elasticity of the customer base at 1.5, the marginal technology progress rate at 7%, and the marginal technology innovation risk at 12%. Model-implied R&D

intensity is especially sensitive to changes in these three parameters.

We show that our model under the aforementioned parameter choices well captures the almost-zero-leverage anomaly. As Table 2 presents, the model generates extremely low leverage, from 1.65% to 3.77%. The corresponding expected R&D value is large (exceeds 50% of firm value) and R&D intensity is high (about from 6.6% to 10%). Such results mean that when operating earnings react to innovation success strongly[24] or the quality of available inventive human capital is high, doing R&D delivers a large value-creation effect on equity. A large R&D value-creation effect naturally accompanies a substantial marginal R&D-value loss due to debt use, which implies an extremely high marginal cost of debt use. Because of such concerns, firms with innovation-sensitive earnings or high R&D efficiency tend to use debt as little as possible (almost zero leverage) and invest in R&D heavily (high R&D intensity). These model implications are supported by related empirical findings. For example, Strebulaev and Yang (2013) and Bessler et al. (2013) document that R&D- intensive firms are more likely to adopt a zero-leverage or almost-zero-leverage policy. Cohen et al. (2013) and Islam and Zein (2020) find that firms and CEOs with better R&D ability will invest more in R&D.

In the present model, firms always adopt an almost-zero or zero leverage policy when R&D intensity is very high. This, however, does not mean that our predictions about zero leverage rely on parameter choice manipulation that generates a counterfactually high R&D intensity. As Table 2 reports, the model-generated R&D intensity at almost zero optimal leverage moderately ranges between 6.6% and 10.1%. Using a large U.S. firm sample, Graham and Leary (2011) present that firms in the lowest quintile of book leverage on average choose the book leverage ratio at 1% and R&D intensity at 36%. Strebulaev and Yang (2013) report a 21.4% mean R&D intensity in their sample of U.S. almost-zero-leverage firms. In our sample of firms with market leverage smaller

[24] The sensitivity of operating earnings under optimal product pricing $\hat{\pi}(\cdot)$ with respect to technology progress can be given by $(\varepsilon-1)\eta^{-1}=[1-\gamma+(\varepsilon-1)^{-1}]^{-1}$, which increases with the price elasticity ε.

than 5%, the median R&D intensity equals 14.9%.[25] Our choice of the price elasticity of the customer base $\varepsilon = 1.5$ also seems moderate. In Hoch et al. (1995) and Foster et al. (2008), the estimated commodity price elasticities typically lie in the range from 1.2 to 3.5. We further test the robustness of our results on almost-zero leverage along with various parameter combinations

Ranges of indicated parameters	Indicated-parameter choices				
	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
τ (20% to 40%)	1.467e-04	2.101e-04	2.826e-04	3.637e-04	4.529e-04

and present the results of robustness tests in Table 3. We show that if the parameters involving R&D efficiency are calibrated to make model-implied R&D intensities match the median R&D intensity in our almost-zero-leverage firm sample, the optimal market leverage ratios will be consistently lower than 1%. These outcomes justify the robustness of our results.

Table 3: Model Predictions about Almost-Zero Leverage Phenomena.

[25] To empirically estimate the R&D intensity of almost-zero-leverage and zero-leverage firms, we construct a U.S. firm sample covering the period from 1950 to 2018.

α (20% to 40%)	3.131e-04	3.129e-04	3.128e-04	3.127e-04	3.126e-04
β (10% to 50%)	3.516e-04	4.101e-04	4.578e-04	5.180e-04	1.253e-03
γ (0.4 to 0.6)	6.408e-03	4.004e-03	2.181e-03	1.001e-03	3.129e-04
ε (1.2 to 1.3)	1.646e-03	7.195e-04	3.129e-04	1.347e-04	6.079e-05
μ_q (1% to 2%)	8.801e-04	6.247e-04	3.988e-04	2.009e-04	6.153e-05
σ_q (20% to 30%)	1.923e-03	9.376e-04	4.541e-04	2.427e-04	1.681e-04
Du (0 yr to 1 yr)	3.183e-04	3.152e-04	3.122e-04	3.093e-04	3.074e-04

Table 3 reports the robustness of numerical predictions with respect to almost-zero leverage along with various parameter combinations. The numbers show optimal market leverage ratios. Except for marginal technology innovation risk and indicated parameters, model parameters are set at their baseline levels. We calibrate marginal technology innovation risk to make model-generated R&D intensity at optimal leverage match the median R&D intensity in our sample of almost-zero and zero leverage firms (about 14.9%). Our sample includes U.S. non-financial unregulated R&D firms with market leverage of smaller than 5%. The sample covers the period from 1950 to 2018.

C. Complexity of the leverage-wage relationship

Firms' involvement in R&D and innovation inevitably incurs costs for financing inventive human capital. Proposition 3 shows endogenous wages (human capital costs) as a joint function of available R&D benefit $V^{Performance}$ and the employment contract value multiplier V^{Multi}. The influence of debt usage or leverage changes on wages is decomposed into the HCL effect and the RC effect. The former (latter) describes the case where changing leverage negatively (positively)

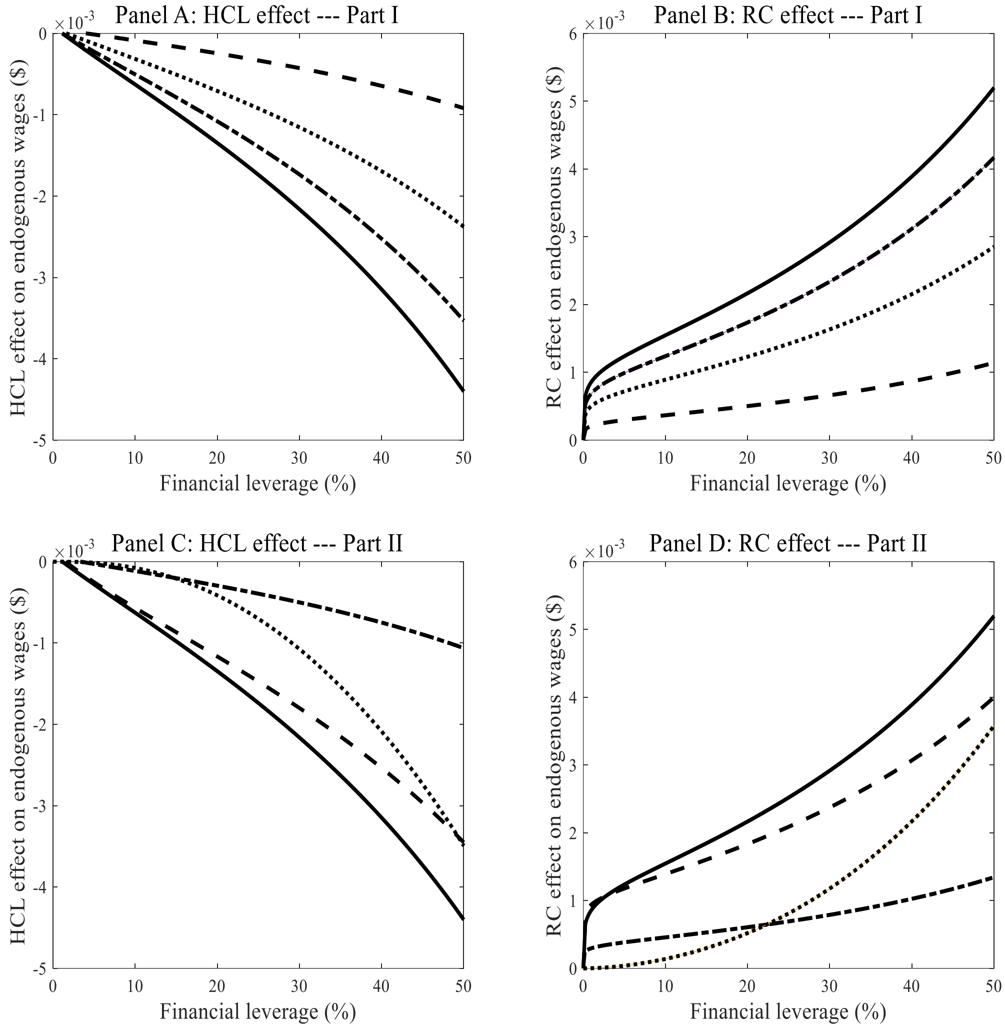


Figure 2: Risk compensation effect and human capital loss effect. In Panels A and B the solid, dashed, dotted, and dash-dotted lines are respectively plotted using the baseline parameters, $\Lambda_{\mu}=3\%$, $\Lambda_{\sigma}=20\%$, and $\beta=4\%$. In Panels C and D the solid, dashed, dotted, and dash-dotted lines are respectively plotted using the baseline parameters, $\mu_q=0.8\%$, $\sigma_q=15\%$, and $\varepsilon=1.1$.

influences wages through available R&D benefit (the employment contract value multiplier). The implications of the HCL and RC effects have been described in Introduction and Section 3C, so that we do not repeat them here. This section aims at (i) numerically verifying the prediction from Proposition 3 about the directions of these two effects, and (ii) examining how the model shapes the endogenous interaction between *optimal* leverage and wages.

Figure 2 plots the HCL effect $\frac{\partial I^*}{\partial V^{Performance}} \times \frac{\partial V^{Performance}}{\partial D}$ (the lines in Panels A and C) and the RC effect $\frac{\partial I^*}{\partial V^{Multi}} \times \frac{\partial V^{Multi}}{\partial Q_d(D)} \times \frac{\partial Q_d(D)}{\partial D}$ (the lines in Panels B and D) as a function of leverage by using various parameter combinations. The HCL effects are negative, while the RC effects are positive. Increasing leverage intuitively enhances these two effects. Also, we can observe that these two effects are both sensitive to R&D efficiency and the features of product demand. When R&D efficiency is lower (the dashed and dotted lines in Panels A and B), product demand is less sensitive to the product price (the dash-dotted lines in Panels C and D), product demand is more stable (the dotted lines in Panels C and D), or product demand grows slower (the dashed lines in Panels C and D), the two effects are less pronounced and display weaker reactions to leverage use changes. In comparison with the solid lines, the dashed, dotted, and dash-dotted lines are flatter. Such results are attributed to the fact that the RC and HCL effects have a strict positive relationship with available R&D value and marginal R&D-value loss due to debt use, respectively. These two R&D-value-related quantities increase with R&D efficiency, product demand volatility, and price competition, but decrease with product demand growth.

Next, we consider the interaction between wages and optimal leverage. As shown in Figure 3, the shapes of the interaction are complex, depending on the trade-off between the HCL effect and the RC effect.[26] This complexity arises from simultaneous endogeneity —changes in exogenous factors or firm characteristics simultaneously but asymmetrically influence endogenous wages

[26] After introducing inventor mobility into the model, we find the endogenous leverage-wage relation along mobility intensity to be an inverted U. For further discussions, see Section 6C.

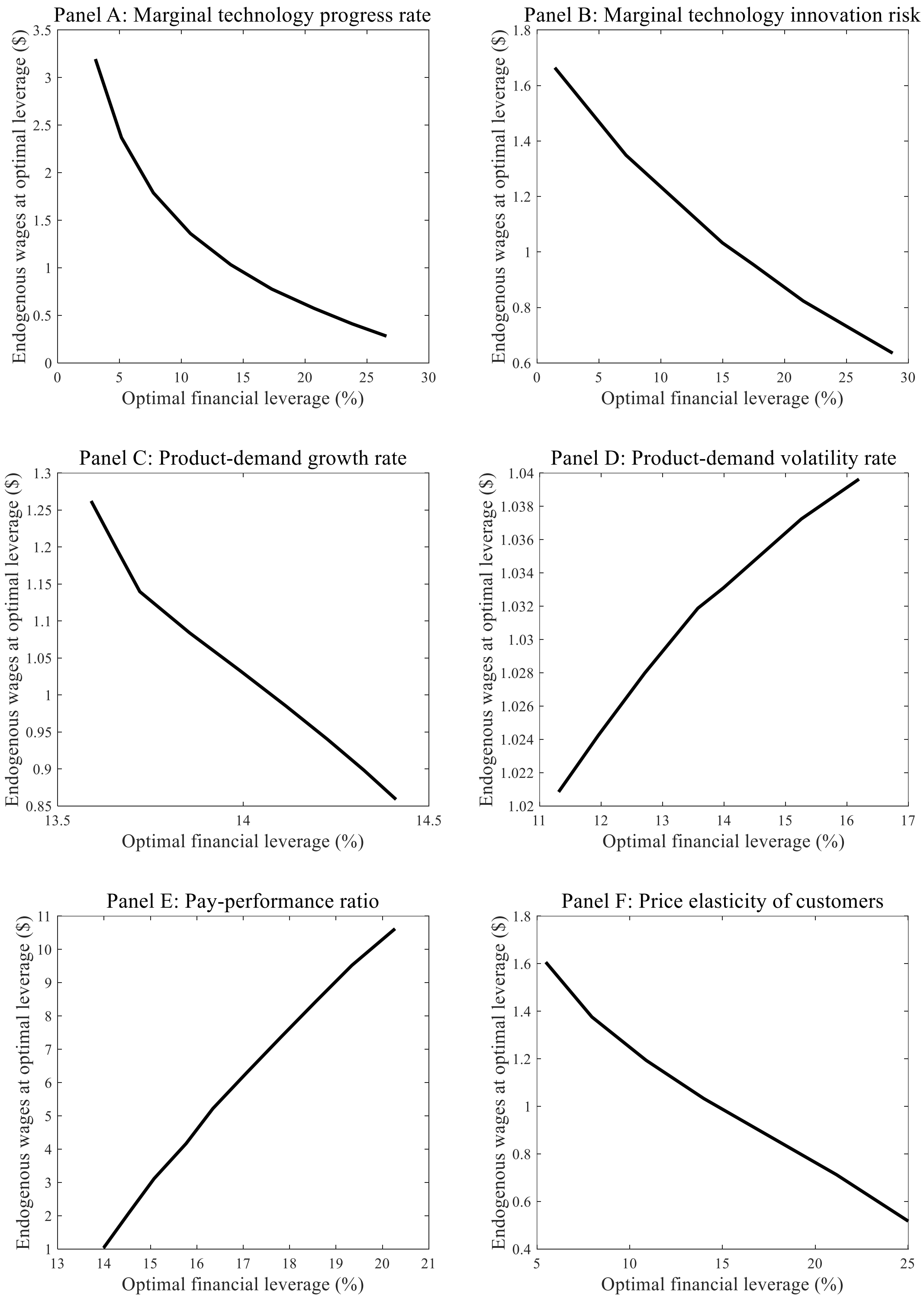


Figure 3: Interactions between endogenous wages and optimal leverage. The lines depict the leverage-wage endogenous interaction along Λ_μ (from 3% to 7%), Λ_σ (from 14% to 20%), μ_q (from 1.2% to 2%), σ_q (from 24% to 30%), β (from 5% to 50%), and ε (from 1.1 to 1.4) in Panels A-F, respectively.

and optimal leverage. We take Panel A as an example for demonstration. Decreasing the marginal technology progress rate lowers the expected value of R&D and marginal R&D-value loss due to debt use, so that firms tend to raise leverage. Such an increase in optimal leverage simultaneously but asymmetrically generates the HCL and RC effects on wages. Since the HCL effect is relatively stronger, marginal decrements in wages due to the HCL effect outweigh marginal increments due to the RC effect. As a result, wages and optimal leverage have an inverse endogenous interaction along with changes in the marginal technology progress rate. Their interactions are inverse under most circumstances. Panels D and E present opposite results. The reason is that in these two panels, the RC effect generated by leverage use adjustments outweighs the corresponding HCL effect.

Our results on the leverage-wage relationship can be thought of as an integration of the results of Berk et al. (2010) and Michaels et al. (2019). Under labor market competitive equilibrium, Berk et al. solve market wages that maximize laborers' expected utility. Raising debt use makes the timing of bankruptcy earlier, and lowers laborers' total expected utility by reducing the expected duration of employment. Firms choosing higher leverage should pay laborers more to compensate them for bearing the unemployment risk that accompanies firm bankruptcy. Such an insight is consistent with the implication of our RC effect. In their model, the total expected utility of market wages, which reflects the value of laborers' outside job options, corresponds to the value of inventors' employment contract V^{Pay} in our model. Moreover, productivity is employed as the underlying state variable of laborers' indirect utility, and therefore, dynamic adjustments in market wages only depend on the current state of productivity, unaffected by debt use. This makes their model unable to isolate the *performance-side* influence of debt use changes on wages (if we interpret productivity as laborers' performance). Their analysis cannot capture the implication of the HCL effect in our model.

Under the surplus-sharing arrangement framework, Michaels et al. (2019) solve endogenous wages by using dynamic bargaining equilibrium. Their central conclusion is that because the total expected contribution of laborers to the firm's cash flows decreases with leverage, raising leverage enables the firm to improve the position of bargaining with laborers. Following the bargaining rule, firms adjust wages downward as leverage increases. The role of laborers' cash-flow contribution corresponds to inventors' innovation-benefit contribution $V^{Performance}$ in our model, so that their conclusion is conceptually similar to our HCL effect. The model of Michaels et al. (2019) has two features that make them ignore the *pay-side* influence of debt changes on wages. First, laborers can always receive full promised wages from the firm, no matter whether it subsequently defaults. Second, laborers are assumed to have the ability to perfectly insure their idiosyncratic wage risks that stem from productivity shock by accessing government risk-transfer programs or relying on other household members' wage earnings. In their model, laborers thus have no incentive to do bargaining over compensation for bearing the idiosyncratic risk on wage earnings.

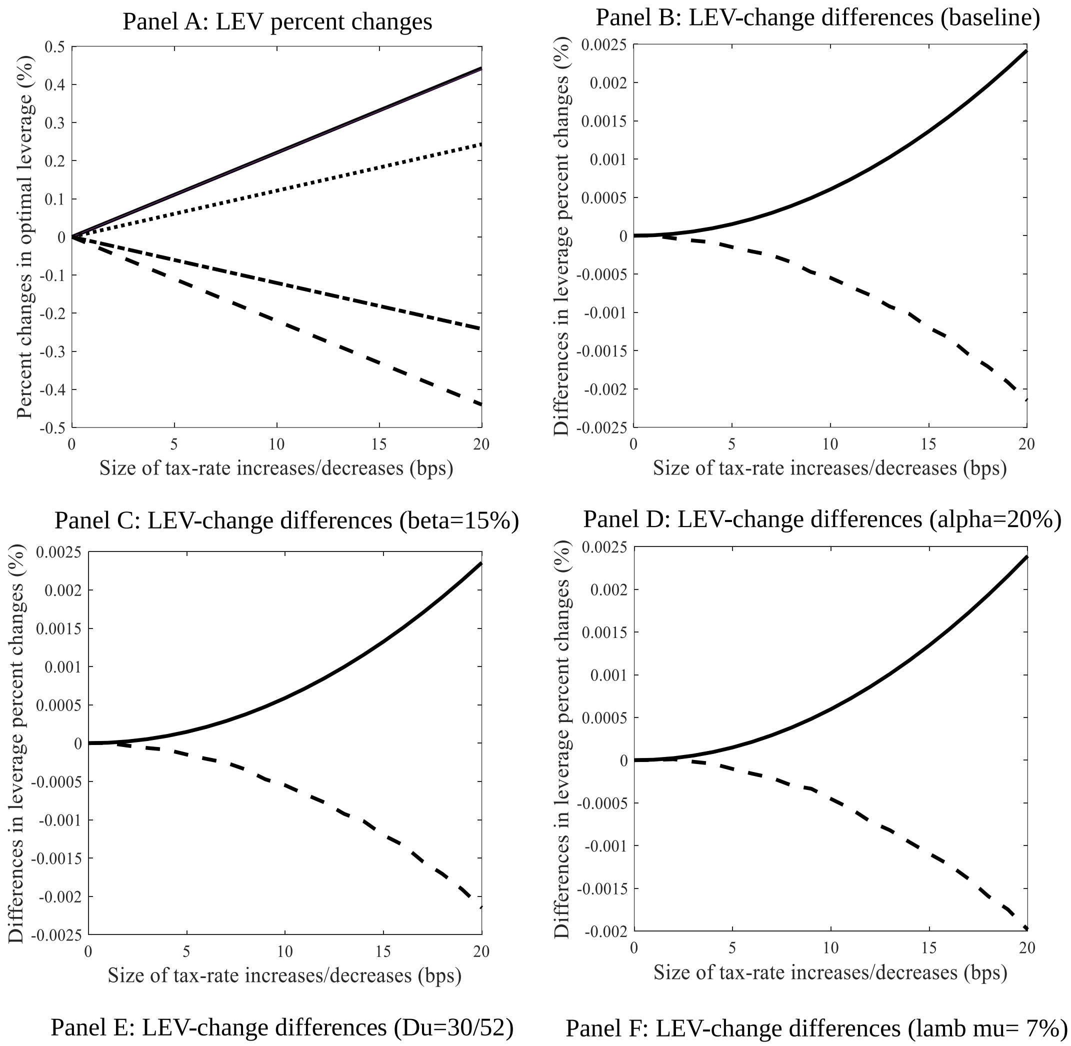
The predictions from Berk et al. (2010) and Michaels et al. (2019) about the leverage-wage relationship are both partly supported by empirical studies. The negative bargaining effect of leverage on wages has been documented in Matsa (2010) and Michaels et al. (2019). Also, Hanka (1998) finds debt is inversely associated with wages in the cross-section. Evidence from Chemmanur et al. (2013) and Akyol and Verwijmeren (2013), however, supports the risk compensation hypothesis of Berk et al. (2010). Agrawal and Matsa (2013) find that laborers' compensation for unemployment shock increases with firm leverage. In contrast with Berk et al. (2010) and Michaels et al. (2019), our model simultaneously captures these two sorts of empirical regularities, and thus, helps settle the debate on the cross-sectional relationship between leverage and laborers' wages or compensation.

Our results rely on three important model features. First, the employment contract value and the value of innovation/R&D benefits contributed by inventors can be measured separately. The reactions of these two economic quantities to debt use changes, hence, can be separately measured as well. Second, we consider a fixed pay-performance ratio that embodies the value-sharing rule between inventors and shareholders as well as inventors' bargaining power, which exogenizes the influence of labor market competition and bargaining behavior on employment contract design as a given requirement for employment, and delivers tractability in solving endogenous wages under partial equilibrium. Third, the pricing formula for employment contracts permits the explicit separation between the value multiplier and wage payments. This separation enables us to express endogenous wages as a simple linear combination of the value multiplier and innovation (or R&D) benefit extracted from inventors. Because of the three model features above, we can easily achieve the distinction between the performance-side effect of leverage adjustments on wages (HCL effect) and the pay-side effect on wages (RC effect).

In summary, we show that the trade-off between the RC and HCL effects plays an important mechanism through which our structural model identifies the cross-sectional relationship between wages and corporate leverage. Varying firm characteristics simultaneously affects the outcomes of leverage decisions and the aforementioned trade-off. Along with characteristic changes, this simultaneity shapes the endogenous interaction between optimal leverage and wages. Their interaction will be positive (negative) when the RC (HCL) effect generated by leverage adjustments outweighs the corresponding HCL (RC) effect. The complexities of such results settle the debate on the leverage-wage relationship in the empirical literature.

D. Asymmetric tax effect on firm innovation and leverage

As argued by the trade-off theory, government guides firms' debt policy by adjusting the tax rate and changing the value of tax benefits on debt. Marginal tax benefits due to debt increases are concave in the tax rate; so that optimal leverage is more sensitive to tax cuts than to tax rises (see the dashed line in Panel B of Figure 4). However, such a theoretical prediction is refuted by Heider and Ljungqvist (2015), who document that the tax-cut effect on leverage is little and weaker than



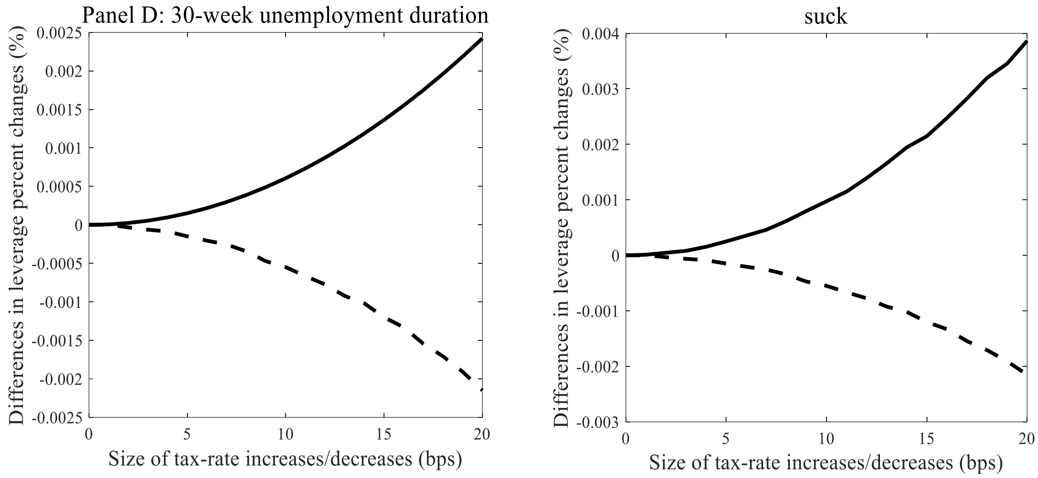


Figure 4: Comparisons of percent changes in optimal leverage between tax rises and tax cuts. In Panel A, the solid (dashed) and dotted (dash-dotted) lines plot percent increases (decreases) in optimal leverage due to tax rises (tax cuts) against the sizes of tax changes by using the baseline model and the benchmark model, respectively. Panels B-F plot percent increases in the optimal leverage ratio due to tax rises minus the corresponding percent decreases due to tax cuts. The solid and dashed lines are plotted using the basic and benchmark models, respectively.

the tax-rise effect. Mukherjee et al. (2017) correspondingly find the asymmetric reactions of R&D investment and innovation outputs to tax changes. Tax rises impede innovation, while tax cuts cause a little effect that boosts innovation later.[27] Because R&D intensity and leverage have a robust monotonic relationship, we infer that there exists a common factor that shapes asymmetries in their tax sensitivities. We now explore the implications of this factor.

In the benchmark model, percent changes in optimal leverage due to tax cuts are larger than those due to tax rises. Such results are contrary to ours. As the solid lines in Panels B-F of Figure 4 show, optimal leverage in our model reacts more to tax rises than to tax cuts.[28] This highlights

[27] Contrary to empirical results in Mukherjee et al. (2017), Atanassov and Liu (2020) document that corporate income tax cuts laggedly and significantly stimulate corporate innovation. A possible reason is that the data sample used by Atanassov and Liu (2020) only includes the events of large tax-rate changes (at least exceed 100 basis points). Such a sample selection criterion could inflate the statistical significance of tax cut effects on innovation. The samples used by Heider and Ljungqvist (2015) and Mukherjee et al. (2017) are relatively broader. Those two papers never impose any constraint on the size of tax changes when constructing their samples.

[28] The empirical finding of Mukherjee et al. (2017) presents that the sizes of adjustments in the state-level corporate income tax rate are small, and the median of these sizes is about 22 bps. The influence of a 1-unit change in the state -level tax rate on firm value is in fact smaller than that of a 1-unit change in the corporate effective tax rate. This is because in addition to state-level income taxes, the calculations of the corporate effective tax rates should take into account the effects of federal and city-level income taxes, dividend taxes, and other taxes (e.g., property taxes). The elasticity of the corporate effective tax rate to the state-level tax rate is smaller than 1. In view of this fact, we consider a reasonable range of the effective tax rate from τ minus 20 bps to τ plus 20 bps when plotting Figures 4-6. The width

the relevance of R&D investment decisions in analyzing the tax sensitivity of leverage. To further examine how the inclusion of R&D decision twists the shape of the tax sensitivity, we next move our attention to Figure 5.

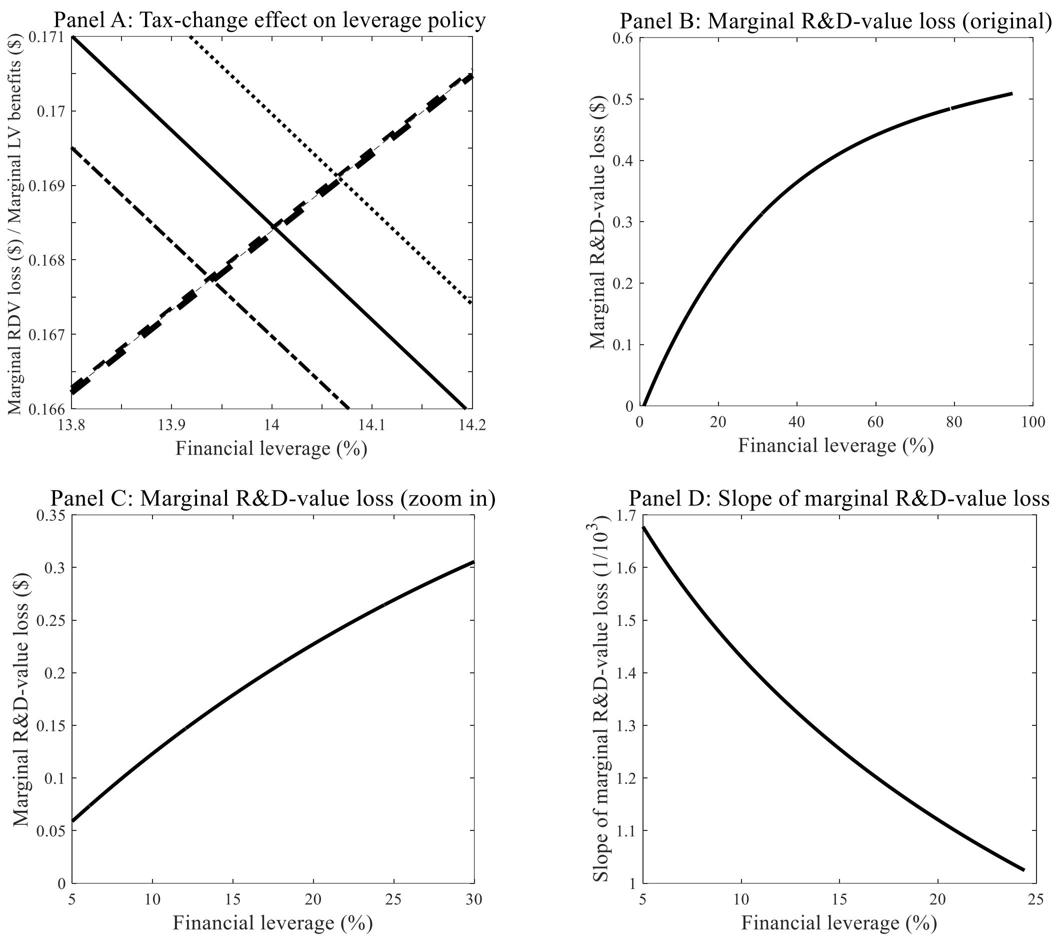


Figure 5: The concavity of marginal R&D-value losses and the tax-change effect on leverage choice optimization. In Panel A, the solid, dotted, and dash-dotted lines plot marginal financial leverage value given $\tau = 32\%$, $\tau = 32.2\%$, and $\tau = 31.8\%$, respectively. The short-dashed, middle-dashed, and long-dashed lines respectively plot marginal R&D-value loss given $\tau = 32\%$, $\tau = 31.8\%$, and $\tau = 32.2\%$ (these three lines almost overlap). Panels B-D plot marginal R&D-value loss and its slope given $\tau = 32\%$. The rest of the parameters are chosen at their baseline levels.

Panel A of this figure depicts marginal leverage value and marginal R&D-value loss against

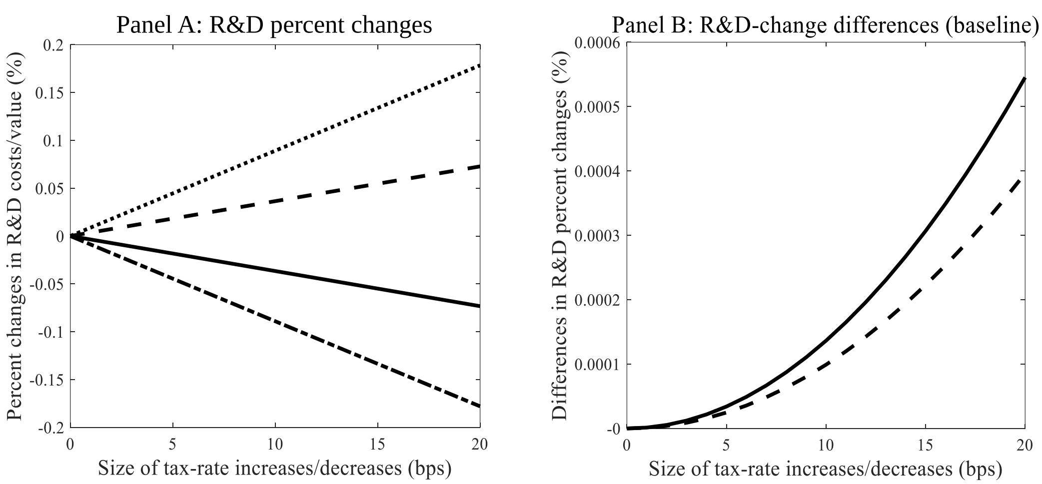
of the interval of effective tax rate changes is calibrated at 20 bps, close to the sample median.

leverage under different tax rates. These patterns show how changes in the tax rate influence the optimal leverage choice via the trade-off between leverage value and R&D value. Tax rises (cuts) raise (lower) debt's tax benefits but lower (raise) after-tax net R&D value, and further result in a rightward (leftward) shift in the curves of marginal leverage value and marginal R&D-value loss. These simultaneous shifts generate a new R&D-leverage value trade-off equilibrium that provides firms a criterion to adjust leverage usage for adapting tax rate changes (recall from Proposition 4 that leverage choice reaches optimization only when marginal R&D-value loss matches marginal leverage value). We observe that marginal leverage value reacts more to tax cuts than to tax rises. This causes the tax cut sensitivity of optimal leverage in the benchmark model to be higher than the corresponding tax rise sensitivity. The size of the shift from the solid line to the dash-dotted line is greater than that from the solid line to the dotted line.

Moreover, in contrast with marginal leverage value, marginal R&D-value loss displays quite weak reactions to tax-rate changes. The short-dashed, middle-dashed, and long-dashed lines almost overlap. The key to twisting the shape of the leverage-to-tax sensitivity, therefore, lies in the *concavity* of marginal R&D-value loss (check Panels B-D), which makes the sensitivity of marginal R&D-value loss to leverage changes strictly decreasing in leverage use. Also, this concavity means that the leverage cut sensitivity of marginal R&D-value loss at any leverage level is higher than the corresponding leverage rise sensitivity.

The occurrence of tax changes leads to the imbalance between marginal leverage value and marginal R&D-value loss. Such an imbalance results from the aforementioned simultaneous but asymmetric shifts in the curves of marginal leverage value and marginal R&D-value loss. The tax-rise (tax-cut) imbalance manifests a case where marginal leverage value at the current leverage level outweighs (is not enough to offset) corresponding marginal R&D-value loss, implying that current leverage downwardly (upwardly) deviates from its optimal level. Firms should offset this imbalance by adjusting leverage usage if they attempt to regain optimal capital structure. Because of the concavity of marginal R&D-value loss, firms will offset the tax-rise (tax-cut) imbalance by increasing (lowering) leverage use in a relatively larger (smaller) size. As a result, the reactions of optimal leverage to tax cuts are weaker than those to tax rises. Our predictions about the shape of the leverage-tax sensitivity are contrary to the benchmark model, but consistent with the empirical findings of Heider and Ljungqvist (2015).

We have been aware of the robust inverse comovement between optimal leverage and R&D-related quantities (e.g., net R&D value and total expected R&D expenditures). A natural question thus arises here —is the asymmetric sensitivity to tax changes a common feature of optimal leverage and R&D policy? The answer to this question can be found in Figure 6. In Panels B-F, the solid (dashed) lines plot percent decreases in net R&D value (total expected R&D expenses) due to tax rises minus corresponding percent increases due to tax cuts. The patterns of these lines consistently show that consistent with the case of leverage, available R&D benefit, and aggregate



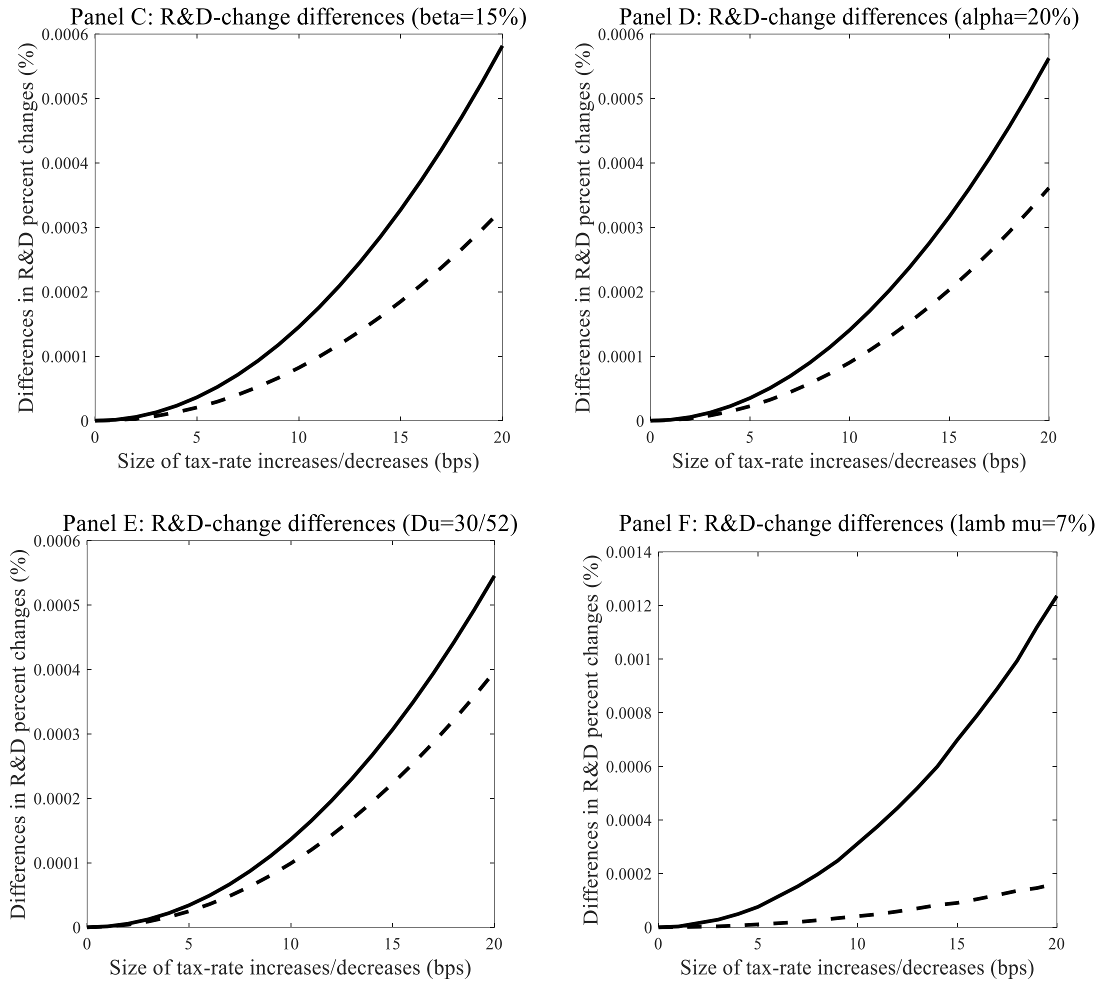


Figure 6: Comparisons of percent changes in R&D policies between tax rises and tax cuts. In Panel A the solid (dotted) and dashed (dash-dotted) lines respectively plot percent changes in net R&D value (total expected R&D expenditures) scaled by firm value due to tax rises and tax cuts. In Panels B-F, the dashed lines (solid lines) plot percent increases in total expected R&D expenditures (percent decreases in net R&D value) scaled by the firm value due to tax rises minus the corresponding percent decreases (percent increases) due to tax cuts.

R&D investment both display stronger reactions to tax rises than to tax cuts. This is because, in the model, optimal R&D investment strictly decreases with leverage (or debt) use, and all R&D-related quantities have a 1-to-1 monotonic relationship with leverage use. As a result, firms' optimal leverage, R&D policies, and R&D-related quantities jointly display asymmetric sensitivities to tax rate changes. Another finding from Figure 6 is that firms' available R&D benefits are negatively related to the tax rate (see the solid line in Panel A). A rise in the tax rate encourages firms to use more debt by increasing the value of debt's tax benefits. As debt use rises, the likelihood of human capital loss due to financial distress increases, and the expected duration of inventor employment shortens. Hence, firms extract a smaller R&D benefit from inventors in place when the tax rate is higher. These implications are in support of empirical evidence from Mukherjee et al. (2017) that tax rises cause firms to acquire fewer innovation outputs (e.g., patents). Asymmetries in the tax sensitivity of R&D policies predicted by our model are in support of Mukherjee et al. (2017) as well. They document that tax rises hinder innovation significantly, but tax cuts boost innovation weakly later. Using the concavity of marginal R&D-value loss, our model offers a partial explanation for this puzzling, stylized fact.

5. Coordination between Innovation Strategies and Product Pricing

While benefits from successful product and technology innovations both increase operating earnings, the mechanisms through which firms absorb these two types of innovation benefits are different. Technology progress motivates firms to sell their products at a lower price, which helps them earn a greater product market share. Because improvements in product quality would attract new customers that drive the growth of product demand, firms absorb product innovation benefits by increasing the product price (Lin, 2012). The coordination between innovation strategy choice and product pricing is crucial for examining the influence of innovation strategy switches on the interaction among leverage, R&D, and product price competition. To explore the implications of this coordination, Section 5 extends the baseline model by allowing for product innovation.

A. The generalized corporate mixed-innovation model

We begin this section by describing the economic setup of product innovation, where the representative firm can continuously discover new product features or functionality from the outcomes of R&D on product innovation. Once new product features are discovered, the firm instantly reformulates its manufacture according to new product features, manufactures new products, and ceases the production of old products (product innovation relies on the manufacture reformulation that generates additional shocks to productivity). The firm, under the zero-inventory condition, does not simultaneously sell new products and old products. The evolution of product features is irreversible. Updating product features might result in product quality improvements (deteriorations) that enhance (reduce) customers' willingness to buy products. The above intuitive notion, similar to Levin and Reiss (1988) and Smolny (1998), inspires us to measure the effect of product innovation on product demand by using the following specification:

$$d\bar{Q}_t \big/ \bar{Q}_t = \tilde{\mu}_q(RD)dt + \tilde{\sigma}_q(RD)dB_t \equiv (\mu_q + \Xi_\mu RD)dt + (\sigma_q + \Xi_\sigma RD)dB_t \qquad (10)$$

where Ξ_μ denotes the product-innovation-related marginal product demand growth rate and Ξ_σ denotes marginal product innovation risk. The specification (10) is also applicable to the case of mixed innovation, because technology innovation does not affect product demand.

According to the above notion and expression (1), we specify the productivity dynamics under mixed innovation by using

$$d\bar{A}_t \big/ \bar{A}_t = \mu_A(RD)dt + \sigma_A(RD)dW_t + \rho\Xi_\sigma RD\,dZ_t \qquad (11)$$

where $\rho \geq 0$ measures the sensitivity of productivity fluctuations to product innovation, and Z is a Brownian motion. Our productivity fluctuation consists of two parts. The first part, $\sigma_A(\cdot)dW_t$, describes the fluctuations arising from the use of new technology generated by R&D on technology innovation (as mentioned in Section 2B). The second part $\rho\Xi_\sigma RD\,dZ_t$ captures the effect induced by product innovation. This is attributed to the intuitive fact that the production of new products must rely on new technology or the reformulation of manufacture. Expressions (10) and (11) respectively reduce the dynamics of product demand and technology in the baseline model, when setting all product-innovation-related parameters to be zero ($\Xi_\sigma = \Xi_\mu = 0$).

The rest of the mixed-innovation model setups are identical to the baseline model, so that we do not describe them repeatedly. To save space, we show technical derivations and the pricing formulas for corporate securities under mixed innovation in online Appendix B4.

B. Model implications

This section performs numerical analysis on the model developed in Section 5A. We consider three types of innovation strategy: technology innovation (given $\Xi_\sigma = \Xi_\mu = 0$, $\Lambda_\sigma \Lambda_\mu \neq 0$), product innovation (given $\Lambda_\sigma = \Lambda_\mu = 0$, $\Xi_\sigma \Xi_\mu \neq 0$), and mixed innovation (given $\Lambda_\sigma \Lambda_\mu \Xi_\sigma \Xi_\mu \neq 0$). We choose the baseline level of marginal product innovation risk Ξ_σ at 8%, the product-innovation-related marginal product demand growth rate Ξ_μ at 0.2%, and the sensitivity of productivity fluctuations to product innovation ρ at 50%. Also, we respectively recalibrate μ_q and σ_q at 1.17% and 3.82% to ensure that the cash flow growth (volatility) rate in optimization is targeted at roughly 2.5% (25.5%). The baseline values of the rest of the parameters remain unchanged.[29]

B.1. Technology innovation versus product innovation

We first make a comparison between two innovations by putting a special focus on product pricing. Recall from Proposition 1 that the optimal product price is strictly increasing in product demand but decreasing in technology. Therefore, firms experiencing technological progress tend to reduce the product price and pursue a greater product market share. If product innovation success boosts product demand, firms will raise the product price but earn a smaller market share. Such a view on the coordination between innovation strategy and product pricing is in line with

[29] We do not discuss the interaction among optimal leverage, R&D investment, and endogenous wages in product-innovation firms, because related results are identical to those in technology-innovation firms.

Kogan et al. (2017) and others, and can be validated by the patterns in Panels A and B of Figure 7.[30]

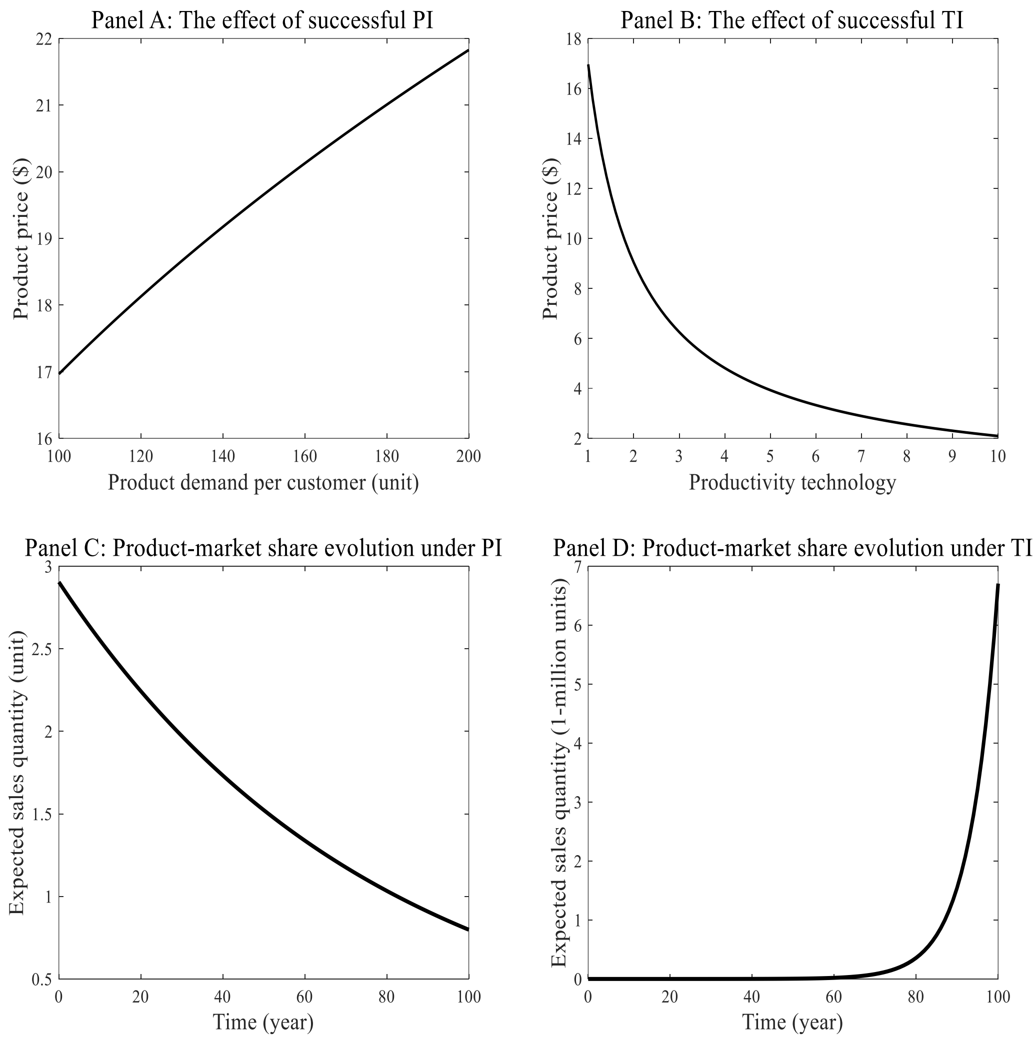


Figure 7: Comparisons of product pricing strategies between product innovation and technology innovation. By considering the growth of product demand as successful product innovation (PI) and technology progress as successful technology innovation (TI), Panels A and B depict the optimal product price against the success of PI and against the success of TI, respectively. Panels C and D depict the expected quantities of product sales as a function of time.

Moreover, the evolution of expected sales quantities $\hat{Q}_D(p_t^*, Q_t)$ under product innovation

[30] For example, Lentz and Mortensen (2008) argue that product innovation success brings firms market power, and enables firms to set the product prices above the marginal cost of production. Adner and Levinthal (2001) conclude that product price decreases are associated with manufacturing process development, while product price increases are associated with quality development

and technology innovation are dramatically different (Panels C and D). Such results are not beyond our expectations. As shown above, firms absorb product (technology) innovation benefits through raising (lowering) the product price. Sales quantities decrease with the product price, so that expected sales quantities under product (technology) innovation fall (grow) over time. We further find that the pace of expected sales quantities in these two innovation cases are different as well. The growth of sales quantities under technological innovation (especially in long term) seems fast, whereas the decay under product innovation seems slow. The reason lies in the asymmetric effect of innovation on product demand. Technological innovation leaves product demand unchanged, and hence, it boosts sales quantities only through lowering the product price. The case of product innovation is more complicated. Successful product innovation drives product demand growth and motivates firms to raise the price, simultaneously. The product-demand-based positive effect on sales quantities partly offsets the product-pricing-based negative effect, so that the former positive effect alleviates the decay of sales quantities.

B.2. Leverage, R&D, and price competition: The role of innovation strategy

We next examine how innovation strategy switches influence the reactions of firms' debt and R&D policies to price competition changes via product pricing. Following Hackbarth and Miao (2012), we employ the price elasticity of the customer base as a measure of the price competition intensity in product markets.[31]

[31] Hackbarth and Miao (2012) similarly use the price sensitivity of product demand as a measure of product market competition.

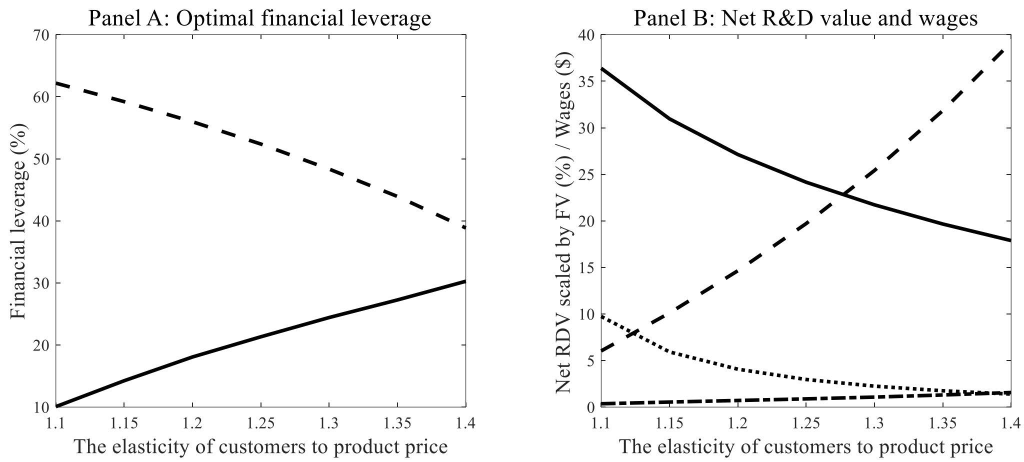


Figure 8: Comparisons between product innovation and technology innovation. Panel A plots optimal leverage under the product (technology) innovation against price elasticity using the solid (dashed) line. In Panel B the solid (dotted) and dashed (dash-dotted) lines depict net R&D value scaled by firm value (endogenous wages) against price elasticity under product innovation and technology innovation, respectively.

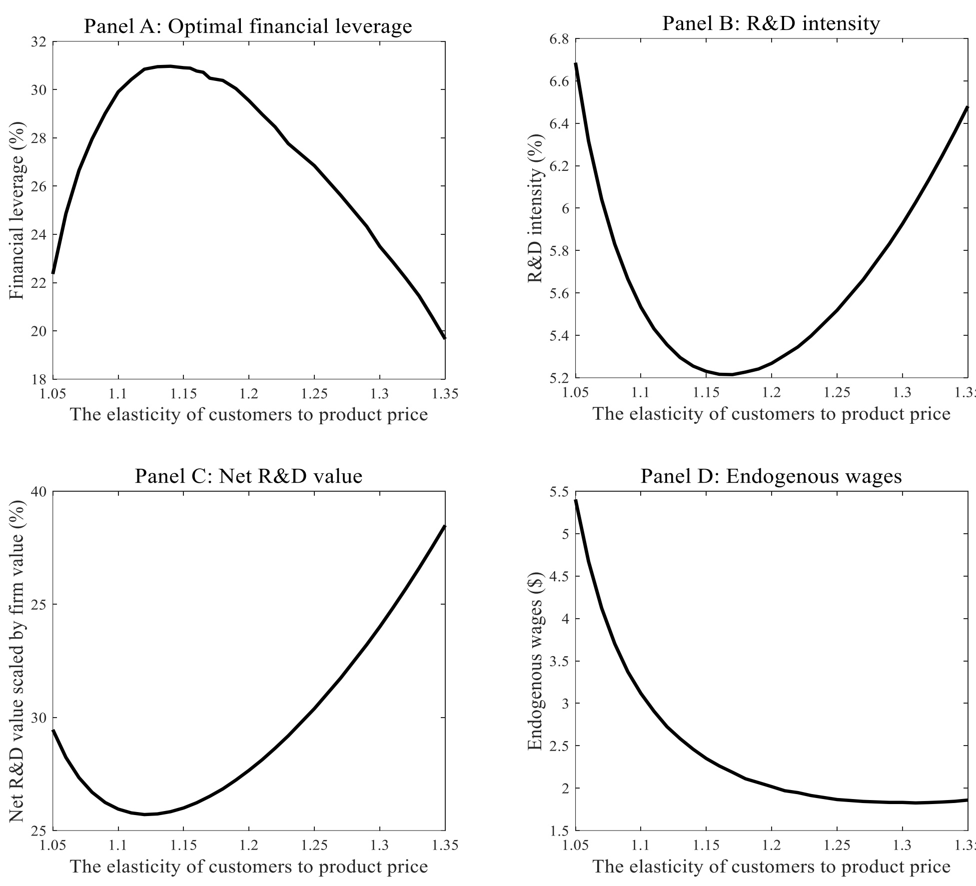


Figure 9: Simultaneous optimization of corporate leverage and R&D policies under mixed innovation. Panels A-D respectively depicts optimal leverage, R&D intensity, net R&D value scaled by firm value, and endogenous wages against price elasticity for a mixed-innovation firm.

According to these Figures 8-9, switching innovation strategy causes structural changes in the leverage-competition relationship. The relationship is an inverted U for a mixed-innovation firm (Panel A of Figure 9), positive for a product-innovation firm (the solid line in Panel A of Figure 8), and inverse for a technology-innovation firm (the dashed line in Panel A of Figure 8). The complexity of this relationship is attributed to the coordination between innovation strategy and product pricing. To absorb technology (product) innovation benefits and convert such intangible benefits into tangible operating earnings, firms adopt a policy of reducing (increasing) the product price. The sensitivity of operating earnings under optimal product pricing to technology and product demand respectively take the form of $\frac{\varepsilon-1}{\eta}=\frac{1}{1-\gamma+(\varepsilon-1)^{-1}}$ and $\frac{1}{\eta}=\frac{1}{\varepsilon(1-\gamma)+\gamma}$ (see Proposition 1). Hence, increasing (lowering) product price competition ε amplifies (dilutes) the benefits of technological progress on earnings, but dilutes (amplifies) the benefits of product demand growth on earnings.

The arguments above mean that product price competition kindles (limits) firms' willingness to do R&D on technology (product) innovation. As implied by the trade-off equilibrium in Proposition 4, a rise (reduction) in firms' willingness to do R&D crowds out (boosts) their debt use. Hence, as product price competition increases, technology (product)-innovation firms expand (reduce) R&D investment and lower (raise) leverage. Mixed-innovation firms' leverage particularly has an inverted-U response to price competition changes. When price competition is moderate (weak or tough), R&D benefits on mixed innovation are relatively small (large), firms' incentives to invest in R&D are weak (strong), and corresponding optimal leverage is high (low).

6. Inventor Mobility and Human Capital Risk Management

In the baseline model, firms can unilaterally request inventors to abandon outside job options without offering extra compensation. Wages are fixed at the initial time and never change, even if the continuation value of the employment contract falls in the future. However, the impact of human capital risk due to labor mobility on firm value has been widely documented (Donangelo, 2014). Labor mobility typically arises from the fact that the continuation value of the wage contract is too low or far lower than the value of laborers' outside options. Firms can avoid the departures of skilled laborers by raising wages or providing additional compensation (e.g., stock options). Given this fact, we now extend the baseline model by taking inventor mobility into account. We additionally impose a dynamic wage-based compensation scheme on the employment contract, which captures the financial effect of hedging human capital risk due to inventor mobility.

A. The model allowing for inventor mobility

We begin with modeling inventor mobility. For brevity, we consider the collective departures of inventors in place as an event of inventor mobility. Besides, we replace the outside-option-loss assumption with a new assumption that managers allow inventors to retain their outside job options, which helps ensure that inventors have incentives to voluntarily leave the firm for job-hopping. Under the new outside-option assumption, inventors can switch jobs without suffering any friction (i.e., temporary unemployment). Inventors' collective departures are triggered by the exercise of outside job options, rather than by firm bankruptcy. Managers do not recruit new inventors after old ones leave. R&D investment will be suspended immediately once inventor mobility occurs.

Intuitively, the likelihood of inventor mobility increases as inventors' continuation value (the continuation value of employment contract) falls. The timing of inventor mobility, hence, can be specified using a stopping time on inventors' continuation value. According to equation (7) and Proposition 3, this value monotonically increases with product demand. Therefore, we use product demand as a state variable and define the random time of inventor mobility as

$$T_M := \inf\left(t > 0 : Q_t \leq \boldsymbol{Q}_M(D,\theta)\right)$$

where $\boldsymbol{Q}_M(D,\theta) \equiv \theta \boldsymbol{Q}_d(D)$ is the mobility-triggering threshold and θ is the intensity of mobility subjectively chosen by inventors. Note that $\theta > 1$, otherwise the timing of bankruptcy is always earlier than inventor mobility and inventor mobility has no relevance to capital structure decision making. θ can be further thought of as the degree of inventors' aversion to the debt issued by the firm (since raising debt use lowers their continuation value). High debt aversion level leads to high positive sensitivity of inventor mobility likelihood to debt increases, meaning that inventors' intentions to switch jobs react to debt increases strongly. Hence, when the degree of debt aversion is greater, debt use is more likely to make firms lose inventive human capital before bankruptcy.

To capture the impact of inventor mobility on R&D valuation, we must re-derive the formula for total expected R&D expense by replacing T_d with T_M. Because $T_M \leq T_d$, indirect bankruptcy costs are also replaced with expected losses in intangible asset value due to inventor mobility. For brevity, we delegate technical derivations and the formulas for corporate securities under inventor mobility to online Appendix B5.

B. The model allowing for human capital risk management

We next build a model of dynamic upward wage adjustments to describe the firm's behavior for fully hedging inventor mobility. Following the literature (He, 2011; Ju and Wan, 2012; and Cao and Wang, 2013), we set the value of inventors' outside job options $\boldsymbol{S}$ to be fixed. In equilibrium, this value matches inventors' continuation value V^{CON}. Then, we consider a process $G_{i,t} \equiv \frac{V^{CON}(Q_t; D, I_{i-1})}{\boldsymbol{S}}$ where $V^{CON}(Q_t; D, I_{i-1}) \equiv \mathrm{E}_t \int_t^{T_d} I_{i-1} e^{-r(s-t)} ds$. This process measures the ratio of inventors' continuation value after $i-1$ times of upward wage adjustment to outside option value. Inventors could exercise outside job options when $G_{i,\square} < 1$. Due to such a job-hopping possibility, firms prevent inventive human capital loss by reconsidering the level of wages.

We denote the *i*th wage adjustment time point by $Tw^{i} := \inf\,(\,t > Tw^{i-1} : G_{i,t} \leq 1-\vartheta)$ where ϑ is the job-switching cost rate. Job-switching costs (such as household moving costs, relocation fees, etc.) should be distinguished from job-seeking costs due to temporary unemployment (such as job searching cost, the charge for labor intermediate services, or employment consulting fees). Job-switching costs have important implications for wage adjustment timing. When job-switching costs $\boldsymbol{S}\vartheta$ outweigh job-switching benefits $\boldsymbol{S} - V^{CON}$, inventors cannot increase their wealth through exercising outside job options.[32] In this case, inventors never quit current jobs and nothing motivates managers to adjust wages for retaining inventors. Inventors exercise outside options only when job-switching benefits exceed job-switching costs. Managers can completely avoid inventors' departures by choosing the wage adjustment timing at an ideal time point at which job-switching costs equal job-switching benefits, i.e., $\boldsymbol{S} - V^{CON}(Q_{Tw^{i}}; D, I_{i-1}) = \boldsymbol{S}\vartheta \Rightarrow G_{i,Tw^{i}} = 1-\vartheta$. The implications above rationalize the definition of random wage adjustment time.

In the model, the role of the job-switching cost rate is to help us determine the size of wage adjustments. At each of the wage adjustment time points, managers reset wages according to their competitive level that ensures the match between outside option value and inventors' continuation value. This means that at the time point just after wage adjustment, the ratio of continuation value to outside option value equals 1; i.e., $\forall i \in \mathbb{N} : G_{i,Tw_{+}^{i-1}} = 1$. When wage adjustment happens, this ratio satisfies the termination condition: $\forall i \in \mathbb{N} : G_{i,Tw^{i}} = 1-\vartheta$. These two conditions jointly imply $V^{CON}(Q_{Tw^{i}}; D, I_{i-1})(1-\vartheta)^{-1} = \boldsymbol{S} = V^{CON}(Q_{Tw_{+}^{i}}; D, I_{i})$, which helps

[32] We simply assume that inventors incur a proportional cost when exercising outside options to switch jobs. Hence, this implies job-switching costs as a fraction of outside option value. Job-switching benefits can be conceptualized as increments in wealth due to the exercise of outside job options, equaling outside option value minus the continuation value of the current employment contract.

us solve the endogenous growth rate of wages: $I_i / I_{i-1} = (1-\vartheta)^{-1} \equiv \phi^{-1}$.

There is a 1-to-1 monotonic relationship between $G_{i,}$ and product demand, which enables us to transform the definition of Tw^i into the following:

$$Tw^i := \inf(t > 0 : Q_t \leq \boldsymbol{Q}_W^i(D,\phi))$$

where $\boldsymbol{Q}_W^i(D,\phi) \equiv \left\{1-\phi^i\left[1-Q(\boldsymbol{Q},\boldsymbol{Q}_d(D),x,y)\right]\right\}^{-1/(x+y)} \boldsymbol{Q}_d(D)$. Note that because $0<\phi<1$ and $\lim_{i\uparrow\infty} \boldsymbol{Q}_W^i(D,\phi) \to \boldsymbol{Q}_d(D)$, as the time of wage adjustment approaches infinity, the timing of wage adjustment will be close to the time point of bankruptcy. This ensures that (i) the range of employment contract value is bounded, and (ii) firms no longer adjust wages after bankruptcy.

We define the (before-tax) cost of hedging inventor mobility as:

$$V^{HIM}(\boldsymbol{Q};D) \equiv \mathrm{E}_{0+} \sum_{i=1}^{\infty} [V^{CON}(Q_{Tw_+^i};D,I_i) - V^{CON}(Q_{Tw^i};D,I_{i-1})]\, \mathrm{e}^{-rTw^i}$$ [33]

where it is measured as the total present value of future spending for retaining inventors in place. Such spending is quantified using increments in inventors' continuation value due to wage adjustment. The formula and technical derivation for hedging costs are given in online Appendix B6. Plugging hedging costs into (8), we obtain a modified version of the capital structure decision problem from which we solve the corresponding optimal debt policy.

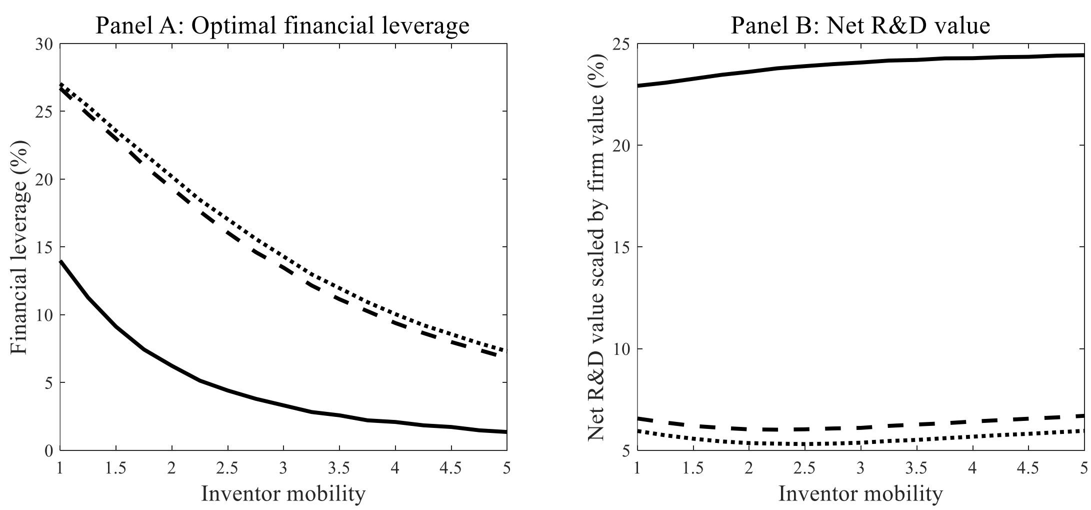
C. Implications of inventor mobility

This subsection quantitatively examines the implications of inventor mobility for firms' debt and R&D policies. For illustration, we plot market leverage, net R&D value, endogenous wages, and the mobility threshold against inventor mobility intensity θ in Figure 10.

[33] Our full-hedge model does not allow us to study the optimization of hedge policy. To deal with issues on optimal hedge, a partial hedge setting (firms do hedge only when net R&D value remains positive) is required. Because the interaction between optimal hedge and leverage is beyond the scope of our paper, we leave this interesting extension to future research.

C.1. Optimal leverage

Firms reduce leverage as inventor mobility intensity increases. In the model, when mobility intensity is higher, debt increases are more likely to invite inventor departures (i.e., higher inventor mobility likelihood), because the mobility threshold is more sensitive to changes in the bankruptcy threshold $\frac{\partial Q_M(D,\theta)}{\partial Q_d(D)} = \theta$. Such results can be thought of as inventors' debt aversion, which is attributed to the negative influence of financial distress risk on their continuation value. Hence, allowing for inventor mobility makes firms bear a higher risk of human capital loss due to debt use, and further exacerbates the detrimental effect of debt on available R&D benefits. Firms suffering the threat of inventor mobility tend to adopt a relatively more conservative debt policy.



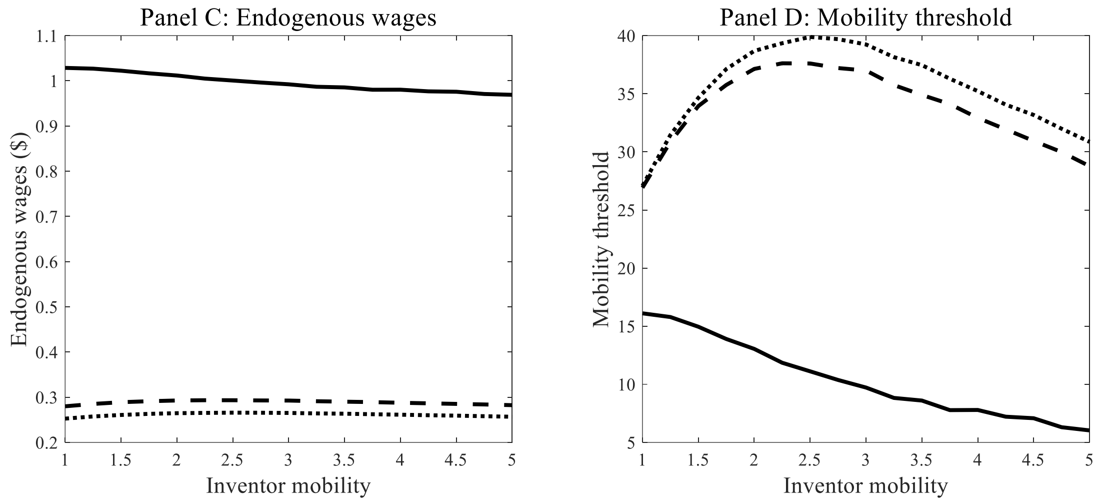


Figure 10: Simultaneous optimization of debt and R&D policies under human capital risk due to inventor mobility. In all panels, the solid lines are plotted using the baseline parameters. The dashed and dotted lines are plotted using $\Lambda_{\mu} = 3\%$ and $\Lambda_{\sigma} = 30\%$, respectively (the rest of the parameters are chosen at their baseline levels).

Our results partially capture empirical phenomena related to key human capital. Israelsen and Yonker (2017) identify human capital concentrated in a few irreplaceable employees (e.g., highly-educated inventors) as *key human capital*. They document that firms possessing key human capital bear the risk arising from key employee departures and take smaller leverage than firms without key human capital. In our model, inventive human capital can be thought of as one type of key human capital. Therefore, inventor mobility is equivalent to key human capital risk arising from inventor departures. Our model offers an innovation-based explanation for the inverse relationship between key human capital risk and leverage. We characterize inventors' (key employees') debt aversion using the mobility intensity parameter (the intensity of key human capital risk). When the degree of debt aversion is greater, increasing debt is more likely to motivate inventors to switch jobs. This further implies that (i) debt use is more likely to make firms lose innovation benefit attached to key human capital; and (ii) the marginal R&D-related cost of debt use, measured as marginal R&D-value loss due to debt use, is higher. Therefore, firms bearing a higher key human capital risk are reluctant to use debt.

C.2. Available R&D benefits and inventor mobility likelihood

Now we consider the implication of inventor mobility for available R&D benefits. According to Panel B, net R&D value has a complex relationship with inventor mobility intensity. When R&D efficiency is set at a relatively high level (the solid line) and a relatively low level (the dotted and dashed lines), this relationship has a positive direction and a U shape, respectively. The value of R&D earned by a firm is, in effect, depends on how long the duration of inventor employment is. Hence, before explaining the aforementioned complex relationship, we must understand how mobility intensity influences the likelihood of inventor mobility. For ease in the subsequent discussion, we plot the mobility threshold as a function of mobility intensity.

Panel D presents that the patterns of the mobility threshold are opposite to the net R&D value. When R&D efficiency is high (low), the mobility threshold has an inverse (inverted-U) relationship with mobility intensity. Our model specifies this threshold with $\boldsymbol{Q}_M(D^*,\theta) \equiv \theta \boldsymbol{Q}_d(D^*)$, so that the influence of mobility intensity changes on it consists of two parts:

$$\frac{\partial \boldsymbol{Q}_M(D^*,\theta)}{\partial \theta} = \boldsymbol{Q}_d(D^*) + \theta \times \frac{\partial \boldsymbol{Q}_d(D^*)}{\partial D^*}\frac{\partial D^*}{\partial \theta} \tag{12}$$

where the first term on the right-hand side represents the positive direct effect, meaning that inventors directly alter the mobility threshold through their subjective choices for mobility intensity. This effect becomes stronger, when firms choose a higher debt level that implies a higher bankruptcy threshold. The second term represents the negative feedback effect, where the reaction of debt usage to mobility intensity changes feeds back into the mobility threshold via the bankruptcy threshold. The likelihood of inventor mobility increases (decreases) with mobility intensity, when the direct (feedback) effect outweighs the feedback (direct) effect.

Changes in mobility intensity and R&D efficiency both simultaneously affect the aforementioned two effects. The direct effect weakens as R&D efficiency or mobility intensity rises. This is because firms employing inventors with higher mobility intensity or R&D efficiency use less debt which implies a lower bankruptcy threshold. Consistent with the case of the direct effect, the feedback effect has a negative relationship with R&D efficiency. The reason is that optimal debt use under higher R&D efficiency is less sensitive to mobility intensity (in Panel A, the solid line given higher R&D efficiency is flatter than the dotted and dashed lines given lower R&D efficiency). The reactions of the feedback effect to R&D efficiency changes are weaker than those of the direct effect.[34] Besides, the feedback effect becomes stronger when mobility intensity is higher. By summarizing the model implications above, we obtain following implications as follows.

The first refers to the case of low R&D efficiency. In this case, when mobility intensity is at a lower level, firms use more debt and set a higher bankruptcy threshold. The direct effect is sufficient to outweigh the feedback effect, so that the mobility threshold and mobility intensity have a positive relationship. As mobility intensity rises, the direct (feedback) effect gradually weakens (strengthens). This twists the direction of their relationship, because the feedback effect outweighs the direct effect when mobility intensity is within the high range.

The second is about the case of high R&D efficiency. Varying R&D efficiency from a low level to a high level causes the direct effect to be insufficient to outweigh the feedback effect (recall that the negative reactions of the direct effect to R&D efficiency increases are relatively stronger). Such results still hold even if mobility intensity is in the low range. Therefore, when

[34] Our undisclosed tests indicate that the values of the partial derivative of the feedback effect with respect to R&D efficiency parameters are lower than those of the direct effect. Such results always hold no matter which the level of mobility intensity we choose.

R&D efficiency is high, the feedback effect always dominates, and the mobility threshold decreases with mobility intensity.

The implications above help us clarify the complex influence of inventor mobility on R&D value. We have shown that a higher mobility threshold always causes a larger inventor mobility likelihood as well as a shorter expected duration of inventor employment. Available R&D benefits are negatively associated with the mobility threshold. Hence, when R&D efficiency is high (low) and mobility intensity has an inverse (inverted-U) relationship with the mobility threshold, its relationship with net R&D value is positive (has a U shape).

Our results are partially supported by Liu et al. (2017), which document that firms suffering higher inventor mobility acquire more innovation outputs from inventors. Our model predicts that only when inventors possess high R&D efficiency, R&D (or innovation) benefits enjoyed by firms increase with the intensity of inventor mobility.

C.3. Interaction among inventor mobility, leverage, and wages

Now we consider the interaction among inventor mobility, leverage, and wages. The patterns of Panels A and C of Figure 10 jointly imply that optimal leverage and wages have an endogenous complex interaction along with mobility intensity changes. Their interaction is inverted-U when R&D efficiency is low (the dashed and dotted lines), but becomes positive when R&D efficiency is high (the solid lines). These results arise from the fact that changing mobility intensity simultaneously generates the RC and HCL effects through leverage decisions. Increasing mobility intensity makes firms adjust leverage downwardly, and such leverage adjustments deliver an RC (HCL) effect that lowers (raises) wages. When R&D efficiency is high, marginal decrements in wages due to the RC effect outweigh marginal increments due to the HCL effect, and hence, the positive co-movement between optimal leverage and wages appears. When

R&D efficiency is low and mobility intensity remains in the low (high) range, marginal increments (decrements) in wages due to the HCL (RC) effect dominate. These generate an inverted-U relationship between mobility intensity and wages as well as an inverted-U wage-leverage endogenous interaction.

D. Implications of human capital risk management

We now examine the implications of hedging human capital risk due to inventor mobility for corporate leverage and R&D policies. Particularly, we calibrate mobility intensity by equating the mobility threshold $\boldsymbol{Q}_M(D^*,\theta)$ with the first wage-adjustment threshold $\boldsymbol{Q}_W^1(D^*,\phi)$, which enables us to assess no-hedging firm value according to the intensity of hedging (firms choosing a higher wage growth rate ϕ^{-1} spend more on hedging). Increments in firm value due to hedging are employed as our measure of hedging benefits. We report major model outputs in Table 4. Two results immediately stand out.

First, the willingness to hedge human capital risk makes firms raise leverage and derive fewer R&D benefits from inventors. This is consistent with the literature on corporate risk management, which finds that hedging lowers default likelihood by smoothing cash flows and permits greater leverage (e.g., Leland, 1998; or Campello et al., 2011;). The existing literature typically considers interest rate risk, exchange rate risk, climate risk, or default risk. In contrast, we consider human capital risk due to inventor mobility, and regard wage-based compensations for retaining inventors as hedging costs. These costs dilute equity holders' available R&D benefits as well as marginal R&D-value loss due to debt use. Such a dilution effect boosts the firm's willingness to use debt via the trade-off mechanism between R&D value and leverage value.

Table 4: Effect of Hedging Human Capital Risk on Model Outputs.

The table shows model outputs at the simultaneous optimization of debt and R&D policies. In each panel the columns (from left to right) report market leverage, net financial leverage value scaled by firm value, net R&D value (minus hedging cost) scaled by firm value, total firm value, initial wages, and hedging benefits scaled by firm value. Inventor mobility intensity is calibrated to equate the mobility threshold with the first wage adjustment threshold. The rest of the model parameters are chosen at their baseline levels.

Model type	Major model outputs					
	Market leverage	Net FL value	Net R&D value	Total firm value	Initial wages	Hedging benefits
Panel A: $\phi = 0.2$ (θ is calibrated at 5.873)						
Hedging	4.463%	1.349%	23.656%	\$813.424	\$0.95872	0.255%
No hedging	1.074%	0.340%	24.473%	\$811.352	\$0.96541	-----
Panel B: $\phi = 0.15$ (θ is calibrated at 6.28)						
Hedging	4.457%	1.347%	23.724%	\$814.141	\$0.95871	0.363%
No hedging	0.962%	0.305%	24.494%	\$811.194	\$0.96348	-----
Panel C: $\phi = 0.1$ (θ is calibrated at 6.64)						
Hedging	4.464%	1.349%	23.791%	\$814.895	\$0.95879	0.471%
No hedging	0.877%	0.278%	24.510%	\$811.076	\$0.96191	-----
Panel D: $\phi = 0.05$ (θ is calibrated at 7.08)						
Hedging	4.457%	1.347%	23.896%	\$816.010	\$0.95879	0.624%
No hedging	0.789%	0.251%	24.526%	\$810.952	\$0.96023	-----

Second, hedging benefits are small, no more than 1% of firm value, and will increase with mobility intensity. Hedging is more valuable when human capital risk is higher. We further see that as hedging benefits increase, the positive hedging effect on leverage becomes greater. Although hedging benefits seem limited, this effect on the leverage ratio is significant, around 340 bps to 360 bps. Similarly, Leland (1998) shows that hedging raises leverage by about 580 bps even if its benefits are only 1.5% of firm value.

7. Concluding Remarks

We study how firms' involvement in innovation influences leverage decisions in a unified model in which firms choose leverage, invest in R&D that drives innovation, absorb innovation benefit via product pricing, and pay wages to inventors. Debt use brings financial distress risk that could make firms lose innovation benefit attached to inventors' human capital. The consequence of debt use for available innovation benefit, measured as marginal R&D-value loss due to debt use, discourages the use of leverage.

Varying firm characteristics simultaneously generate the human capital loss effect and the risk compensation effect on wages through leverage decision. This simultaneity will shape an endogenous positive, inverse, or non-monotonic leverage-wage interaction, depending on the trade-off between the aforementioned two effects.

The concavity of marginal R&D -value loss makes firms react more to tax rises than to tax cuts when adjusting towards the target leverage ratio.

We extend the model in human capital risk due to innovation strategy switches and inventor mobility. Innovation strategy switches twist the influence of price competition on leverage. In response to a rise in competition, product (technological) innovation firms raise (reduce) leverage and reduce (expand) R&D investment. In mixed innovation firms, the leverage (R&D)-competition relationship is inverted-U (U). These arise from the coordination between innovation strategy and product pricing — firms absorb product-demand-growth (technology-progress) benefits by raising (reducing) the product prices, so that price competition is unfavorable (favorable) for product (technological) innovation, but alleviates (exacerbates) the detrimental effect of debt on corresponding innovation benefit.

Inventor mobility, which can be thought of as inventors' debt aversion, exacerbates the detrimental effect of debt on available innovation benefits. Firms suffering the threats of inventor mobility take smaller leverage. Hedging human capital risk due to inventor mobility permits greater leverage but hedging costs dilute R&D value earned by shareholders.

Overall, we propose a unified theoretical and tractable model to simultaneously capture several puzzling leverage features in the cross-section, such as the almost-zero-leverage anomaly, the asymmetric leverage-tax sensitivity, and the mixed leverage-wage relationship. We also provide new and empirically testable predictions among corporate leverage, inventor mobility, human capital risk management, and the coordination between innovation strategies and product pricing. Our model nests several existing R&D/innovation models as special cases, and has a rich set of attractive features. It can be further applied to research in corporate finance, asset pricing, endogenous economic growth, and innovation management.

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Online Appendix for “Understanding Leverage: New Insights from Innovation”

16th October 2021

Abstract

This online appendix provides supplemental materials accompanying the paper “Understanding Leverage: New Insights from Innovation”. Appendix A gives a detailed comparison among various R&D/innovation models in the literature and ours. Appendix B shows technical details about the derivation for model solutions.

Keywords: leverage; wages; R&D intensity; innovation strategy; inventor mobility.

JEL classification: G32; J31; O32; J60; O31.

A. Comparison among Various R&D/Firm-Innovation Models

The comparison among various R&D/innovation models includes SSS (Sapra et al., 2014), LM (Lentz and Mortensen, 2016), J (Jones, 1995), BG (Bena and Garlappi, 2019), ABBGH (Aghion et al., 2005), G (Gu, 2016), LP (Lyandres and Palazzo, 2016), AL (Adner and Levinthal, 2001), DS (Dasgupta and Stiglitz, 1980), AS (Athey and Schmutzler, 1995), GG (Goettler and Gordon, 2014), MZ (Malamud and Zucchi, 2019), C (Caggese, 2012), GS (Galasso and Simcoe, 2011), La (Li, 2011), Lb (Lin, 2012), CK (Cohen and Klepper, 1996), LR (Levin and Reiss, 1988), KK (Klette and Kortum, 2004), and ours. To show that our model can be extended by allowing for limited innovation failure tolerance, we offer related technical derivations in online Appendix B7.

	Model endogenous outputs					**Innovation strategy type**					**Other model features**				
Model type	Debt	R&D spending	Product price	Human cost	Innovation Timing	Technology innovation	Product innovation	Mixed Innovation	Unidentified	Separation btw strategies	Inventor mobility	Limited failure tolerance	R&D/innovation uncertainty	Corp. tax	Link btw firm risk and R&D/innovation rate
SSS					●				●				●		●
LM		●	●				●						●		
DS		●	●			●									
J		●				●							●		
BG					●	●							●		●
ABBGH		●				●							●		
G					●				●				●		
LP					●				●				●		
AL			●		●	●	●	●		●					
AS			●		●	●	●	●		●			●		
GG		●	●				●						●		
MZ		●	●	●		●							●		●
C	●				●	●							●		●
GS					●				●				●		
La					●				●				●		
Lb		●				●	●	●		●					
CK		●				●	●	●		●					
KK		●				●							●		●
LR		●	●					●							
Ours	●	●	●	●		●	●	●		●	●	●	●	●	●

B. Technical Derivations

B.1. Derivations for optimal product pricing

According to the expression (2) in Section 2D, earnings flows take the form:

$$\hat{\pi}(p_t, Q_t, A_t) = p_t^{1-\varepsilon} Q_t - \delta p_t^{-\varepsilon/\gamma} Q_t^{1/\gamma} A_t^{-1/\gamma}.$$

Differentiating the earnings function with respect to product price, we obtain the following FOC:

$$\left.\frac{\partial \hat{\pi}}{\partial p_t}\right|_{p_t = p_t^*} = (1-\varepsilon)(p_t^*)^{-\varepsilon} Q_t + \tfrac{\delta\varepsilon}{\gamma}(p_t^*)^{-1-\varepsilon/\gamma} Q_t^{1/\gamma} A_t^{-1/\gamma} = 0,$$

which implies $p_t^* = Q_t^{(1-\gamma)/\eta} A_t^{-1/\eta} (\tfrac{\delta\varepsilon}{\varepsilon-\eta})^{\gamma/\eta}$. The optimal product price is strictly nonnegative, since $\eta = \varepsilon + \gamma - \varepsilon\gamma = \gamma + \varepsilon(1-\gamma) > 0$ and $\varepsilon - \eta = \gamma(\varepsilon - 1) > 0$. Differentiating the FOC with respect to product price, we obtain:

$$\begin{aligned}
\left.\frac{\partial^2 \hat{\pi}}{\partial p_t^2}\right|_{p_t = p_t^*} &= -\varepsilon(1-\varepsilon)(p_t^*)^{-\varepsilon-1} Q_t - \tfrac{\delta\varepsilon(\varepsilon+\gamma)}{\gamma^2}(p_t^*)^{-2-\varepsilon/\gamma} Q_t^{1/\gamma} A_t^{-1/\gamma} \\
&= (\varepsilon-1)(p_t^*)^{-\varepsilon-1} Q_t \left[\varepsilon - (p_t^*)^{\frac{-\eta}{\gamma}} Q_t^{1/\gamma-1} A_t^{-1/\gamma} \tfrac{\delta\varepsilon}{\gamma} \tfrac{1+\varepsilon/\gamma}{\varepsilon-1} \right] \\
&= (\varepsilon-1)(p_t^*)^{-\varepsilon-1} Q_t \left[\varepsilon - (p_t^*)^{\frac{-\eta}{\gamma}} (p_t^*)^{\frac{\eta}{\gamma}} \tfrac{\gamma+\varepsilon}{\gamma} \right] \\
&= (\varepsilon-1)(p_t^*)^{-\varepsilon-1} Q_t (\varepsilon - 1 - \varepsilon\gamma^{-1}).
\end{aligned}$$

Because $\varepsilon \in (1, 1+\gamma^{-1})$, we have $\varepsilon - 1 < \gamma^{-1} < \varepsilon\gamma^{-1}$ and the SOC

$$\left.\frac{\partial^2 \hat{\pi}}{\partial p_t^2}\right|_{p_t = p_t^*} = \underbrace{(\varepsilon-1)(p_t^*)^{-\varepsilon-1} Q_t}_{sign\,(+)} \underbrace{(\varepsilon - 1 - \varepsilon\gamma^{-1})}_{sign\,(-)} < 0,$$

which validates the optimality implied by FOC. This completes the proof.

B.2. Derivations for available R&D benefit

Recall that, from Problem (3), available R&D benefit has follows expression:

$$V^{Performance}(\boldsymbol{A}\;\;, \boldsymbol{Q}; D, RD) \equiv \mathrm{E}_{0_+} \int_{0_+}^{T_d} [\hat{\pi}^*(Q_t, A_t) - \hat{\pi}^*(Q_t, \boldsymbol{A}) - RD]\, \mathrm{e}^{-rt}\, dt.$$

For convenience in derivations, we decompose this expression into the following:

$$V^{Performance} \equiv \mathrm{E}_{0_+} \int_{0_+}^{T_d} [\, \hat{\pi}^*(Q_t, A_t) - \hat{\pi}^*(Q_t, \boldsymbol{A}) \,] \mathrm{e}^{-rt}\, dt - \mathrm{E}_{0_+} \int_{0_+}^{T_d} RD\,\mathrm{e}^{-rt} dt. \qquad \text{(A.1)}$$

The first term on the right-hand side of (A.1) can be further derived as

$$\begin{aligned}
&\mathrm{E}_{0_+} \int_{0_+}^{T_d} \hat{\Pi}(Q_t, A_t)\mathrm{e}^{-rt}\, dt \\
&\quad = \mathrm{E}_{0_+} \int_{0_+}^{\infty} \hat{\Pi}(Q_t, A_t)\mathrm{e}^{-rt}\, dt - \mathrm{E}_{0_+} \int_{T_d}^{\infty} \hat{\Pi}(Q_t, A_t)\mathrm{e}^{-rt}\, dt \\
&\quad = \mathrm{E}_{0_+} \int_{0_+}^{\infty} \hat{\Pi}(Q_t, A_t)\mathrm{e}^{-rt}\, dt - \mathrm{E}_{0_+} \mathrm{e}^{-rT_d}\, \mathrm{E}_{T_d} \int_{T_d}^{\infty} \hat{\Pi}(Q_t, A_t)\mathrm{e}^{-r(t-T_d)} dt = V_I^* - V^{IBC}
\end{aligned}$$

Regarding intangible asset value, according to Section 3A and Ito's lemma, the stochastic differential equations (SDEs) of earnings under optimal product pricing are

$$\begin{aligned}
d\hat{\pi}^*(Q_t, A_t) &= \hat{\pi}_Q^*\, dQ_t + 0.5\hat{\pi}_{QQ}^*(dQ_t)^2 + \hat{\pi}_A^*\, dA_t + 0.5\hat{\pi}_{AA}^*(dA_t)^2 \\
&= \hat{\pi}^*(Q_t, A_t)[\, \mu_\pi(RD)\, dt + \sigma_{\pi B}\, dB_t + \sigma_{\pi W}(RD)\, dW_t \,]
\end{aligned}$$

$$d\hat{\pi}^*(Q_t, \boldsymbol{A}) = \hat{\pi}_Q^*\, dQ_t + 0.5\hat{\pi}_{QQ}^*(dQ_t)^2 = \hat{\pi}^*(Q_t, \boldsymbol{A})[\, \mu_\pi(0)\, dt + \sigma_{\pi B}\, dB_t \,]$$

where $\mu_\pi(RD) \equiv \frac{\mu_q}{\eta} + \frac{\sigma_q^2}{2\eta} \times \frac{(1-\eta)}{\eta} + \frac{(\varepsilon-1)\, RD\, \Lambda_\mu}{\eta} + \frac{(\varepsilon-1)\,(\Lambda_\sigma RD)^2}{2\eta} \times \frac{\varepsilon-1-\eta}{\eta}$, $\sigma_{\pi W}(RD) \equiv \frac{(\varepsilon-1)\, \Lambda_\sigma RD}{\eta}$, and $\sigma_{\pi B} \equiv \frac{\sigma_q}{\eta}$. The forms of the SDEs above are identical to the stochastic dynamics of EBIT in Goldstein et al. (2001). Therefore, using their formula for asset value easily we obtain the explicit solution to initial intangible asset value:

$$V_I^* = A(\boldsymbol{Q}, \boldsymbol{A}\ )K(RD) - A(\boldsymbol{Q}, \boldsymbol{A})K(0), \qquad \text{(A.2)}$$

where the functions A and K are defined as $A(a, b) \equiv a^{1/\eta}\, b^{(\varepsilon-1)/\eta}$ and $K(a) \equiv \Phi\left[r - \mu_\pi(a)\right]^{-1}$, respectively. The constant Φ is $\Phi \equiv \left(\frac{\delta\varepsilon}{\varepsilon-\eta}\right)^{1-\varepsilon/\eta} \frac{\eta}{\varepsilon}$.

Regarding indirect bankruptcy costs due to inventor departures, V^{IBC} is as follows

$$V^{IBC} = \mathrm{E}_{0_+} \mathrm{e}^{-rT_d}\, V_{I,T_d}^* = \mathrm{E}_{0_+} \mathrm{e}^{-rT_d}\, V_{T_d}^* - \mathrm{E}_{0_+} \mathrm{e}^{-rT_d}\, V_{T,T_d}^*. \qquad \text{(A.3)}$$

On the right-hand side of equation (A.3), by using the formula for asset value in Goldstein et al. (2001), we can derive the first term as follows:

$$\begin{aligned}
\mathrm{E}_{0_+} \mathrm{e}^{-r\,T_d} V^*_{T_d} &= \mathrm{E}_{0_+} \mathrm{e}^{-r\,T_d} A\,(Q_{T_d}, A_{T_d})\,K(RD) \\
&= Q^{1/\eta}_{T_d} K(RD)\,\mathrm{E}_{0_+} \mathrm{e}^{-r\,T_d} A^{(\varepsilon-1)/\eta}_{T_d} \\
&= Q^{1/\eta}_{T_d} \boldsymbol{A}^{\ (\varepsilon-1)/\eta} K(RD)\,\mathrm{E}_{0_+} \mathrm{e}^{\{[\Lambda_\mu RD - 0.5(\Lambda_\sigma RD)^2(1-(\varepsilon-1)\eta^{-1})](\varepsilon-1)\eta^{-1} - r\}\,T_d} \\
&= A\,(\boldsymbol{Q}_d(D), \boldsymbol{A}\ )\,K(RD) \int_{0_+}^{\infty} H(t)\, f^{T_d}_{0_+}(t;D)\,dt.
\end{aligned}$$

The second term is equivalent to $\mathrm{E}_0\, \mathrm{e}^{-r\,T_d} D\,(1-\tau)^{-1}$ because $T_d \equiv \inf\,(t \,|\, (1-\tau)V^*_{T,t} \le D)$. Let $\xi(\boldsymbol{Q};D) \equiv \mathrm{E}_0\, \mathrm{e}^{-r\,T_d} D\,(1-\tau)^{-1}$. According to the Feynman-Kac theorem, it has an explicit solution that satisfies the following ordinary differential equation (ODE):

$$r\,\xi(Q;D) = Q\,\mu_q\,\xi_Q(Q;D) + 0.5Q^2\,\sigma_q^2\,\xi_{QQ}(Q;D)$$

$$\textbf{s.t.}\ \ \lim_{Q\uparrow\infty} \xi(Q;D) = 0 \ \text{ and } \ \lim_{Q\downarrow Q_d(D)} \xi(Q;D) = D\,(1-\tau)^{-1}.$$

Its solution is $\xi(\boldsymbol{Q};D) = D\,(1-\tau)^{-1}\left(\frac{Q}{Q_d(D)}\right)^{-x-y} = V^*_{T,T_d}\,Q\,(\boldsymbol{Q}, \boldsymbol{Q}_d(D), x, y)$ where the function $Q\,(a,b,c,d) \equiv (ab^{-1})^{-c-d}$, $x \equiv \mu_q\,\sigma_q^{-2} - 0.5$, $y \equiv (x^2\sigma_q^2 + 2r)^{0.5}\sigma_q^{-1}$, and $\upsilon \equiv \frac{\varepsilon-1}{\eta}$. The functions $\boldsymbol{Q}_d(D)$ and $H(\cdot)$ are defined in Section 3B. The default density function is given by $f^{T_d}_{0_+}(t;D) \equiv \mathrm{n}\left(\frac{-z(D)+\sigma_q^2 xt}{\sigma_q\,t^{0.5}}\right)\frac{-z(D)}{\sigma_q\,t^{1.5}}$ where $\mathrm{n}(\cdot)$ denotes the standard normal density function and $z(D) \equiv \ln\frac{Q_d(D)}{Q}$. Hence, we have

$$V^{IBC} = A\,(\boldsymbol{Q}_d(D), \boldsymbol{A}\ )\,K(RD) \int_{0_+}^{\infty} H(t)\, f^{T_d}_{0_+}(t;D)\,dt - V^*_{T,T_d}\,Q\,(\boldsymbol{Q}, \boldsymbol{Q}_d(D), x, y). \qquad \text{(A.4)}$$

Regarding total expected R&D expenses (the second term on the right-hand side of (A.1)), the Feynman-Kac theorem implies the implied ODE:

$$rV^{Expense}(Q;D,RD) = Q\,\mu_q\,V^{Expense}_Q(Q;D,RD) + 0.5Q^2\,\sigma_q^2\,V^{Expense}_{QQ}(Q;D,RD) + RD$$

s.t. $\lim_{Q\uparrow\infty} V^{Expense}(Q;D,RD) = RD\,r^{-1}$ and $\lim_{Q\downarrow Q_d(D)} V^{Expense}(Q;D,RD) = 0$.

The explicit solution is given by

$$V^{Expense} = RD\,r^{-1}\left[1 - Q\,(\boldsymbol{Q}, \boldsymbol{Q}_d(D), x, y)\right]. \tag{A.5}$$

Using (A.2) - (A.5), we obtain the formula for available R&D benefit, which completes the proof.

B.3. Derivations for endogenous wages

The net present value of an employment contract is:

$$V^{Pay}(\boldsymbol{Q};D) \equiv \mathrm{E}_{0_+}\int_{0_+}^{T_d} I\,\mathrm{e}^{-rt}\,dt - \mathrm{E}_{0_+}\mathrm{e}^{-rT_d}\int_{T_d}^{T_d+Du} \varphi I\,\mathrm{e}^{-r(t-T_d)}\,dt$$

Using the Feynman-Kac theorem, we obtain the implied ODE:

$$rV^{Pay}(Q;D) = Q\,\mu_q V_Q^{Pay}(Q;D) + 0.5Q^2\sigma_q^2 V_{QQ}^{Pay}(Q;D) + I$$

s.t. $\lim_{Q\uparrow\infty} V^{Pay}(Q;D) = I\,r^{-1}$; and $\lim_{Q\downarrow Q_d(D)} V^{Pay}(Q;D) = \varphi I\,r^{-1}(\mathrm{e}^{-rDu}-1)$.

Because this ODE admits a general solution, the value of the employment contract can be derived as

$$V^{Pay} = Ir^{-1} - Ir^{-1}Q\,(\boldsymbol{Q},\boldsymbol{Q}_d(D),x,y) + \varphi I\,r^{-1}(\mathrm{e}^{-rDu}-1)Q\,(\boldsymbol{Q},\boldsymbol{Q}_d(D),x,y).$$

From the definition of the pay-performance ratio, we know $\beta V^{Performance} = V^{Pay}$ and

$$\beta V^{Performance} = I^*r^{-1} - I^*r^{-1}Q\,(\boldsymbol{Q},\boldsymbol{Q}_d(D),x,y) + \varphi I^*r^{-1}(\mathrm{e}^{-rDu}-1)Q\,(\boldsymbol{Q},\boldsymbol{Q}_d(D),x,y),$$

which implies the explicit form of endogenous wages:

$$I^* = r\beta V^{Performance}\,[1 + (\varphi\mathrm{e}^{-rDu} - \varphi - 1)Q\,(\boldsymbol{Q},\boldsymbol{Q}_d(D),x,y)]^{-1} \tag{A.6}$$

This completes the proof.

B.4. Solving the model allowing for mixed innovation

This appendix solves the model allowing for mixed innovation. Because innovation does not affect financial leverage value and the formula for the optimal product price remains unchanged, we put our focus on the derivations for available R&D benefit and endogenous wages.

Similar to Problem (3) in the main text, we first formulate the expression of available R&D benefit as follows:

$$\bar{V}^{Performance}(\bar{\boldsymbol{A}},\bar{\boldsymbol{Q}};D,RD) \equiv \mathrm{E}_{0_+}\int_{0_+}^{\bar{T}_d}[\hat{\pi}^*(\bar{Q}_t,\bar{A}_t)-\hat{\pi}^*(Q_t,\boldsymbol{A})-RD]\,\mathrm{e}^{-rt}\,dt$$

where $\bar{T}_d := \inf(t>0:(1-\tau)\bar{V}^*_{T,t}\le D)$ denotes the random default time. Following the technique in Appendix B2, we decompose $\bar{V}^{Performance}$ into three parts: (i) intangible asset value, (ii) indirect bankruptcy costs due to inventor departures, and (iii) expected total R&D expenses. That means,

$$\begin{aligned}
\bar{V}^{Performance} &= \bar{V}_I^* - \bar{V}^{IBC} - \bar{V}^{Expense}\\
&\equiv \mathrm{E}_{0_+}\int_{0_+}^{\infty}[\hat{\pi}^*(\bar{Q}_t,\bar{A}_t)-\hat{\pi}^*(Q_t,\boldsymbol{A})]\,\mathrm{e}^{-rt}\,dt\\
&\quad - \mathrm{E}_{0_+}\mathrm{e}^{-r\bar{T}_d}\,\mathrm{E}_{\bar{T}_d}\int_{\bar{T}_d}^{\infty}[\hat{\pi}^*(\bar{Q}_t,\bar{A}_t)-\hat{\pi}^*(Q_t,\boldsymbol{A})]\,\mathrm{e}^{-r(t-\bar{T}_d)}dt - \mathrm{E}_{0_+}\int_{0_+}^{\bar{T}_d} RD\,\mathrm{e}^{-rt}dt
\end{aligned}$$

First, we consider the intangible asset value. Similar to (A.2), it has an explicit solution:

$$\bar{V}_I^* = A(\bar{\boldsymbol{Q}},\bar{\boldsymbol{A}})M(RD) - A(\boldsymbol{Q},\boldsymbol{A})M(0) \tag{A.7}$$

where $\bar{\mu}_\pi(RD) \equiv \frac{\mu_q+\Xi_\mu RD}{\eta} + \frac{(\sigma_q+\Xi_\sigma RD)^2}{2\eta}\times\frac{(1-\eta)}{\eta} + \frac{(\varepsilon-1)RD\Lambda_\mu}{\eta} + \frac{(\varepsilon-1)(RD^2\Lambda_\sigma^2+\rho^2 RD^2\Xi_\sigma^2)}{2\eta}\times\frac{\varepsilon-1-\eta}{\eta}$ and the function M is defined as $M(a)\equiv\Phi\left[r-\bar{\mu}_\pi(a)\right]^{-1}$.

Next, regarding the indirect bankruptcy costs, we have:

$$\begin{aligned}
\bar{V}^{IBC} &= \mathrm{E}_{0_+}\mathrm{e}^{-r\bar{T}_d}(\mathrm{E}_{\bar{T}_d}\int_{\bar{T}_d}^{\infty}\hat{\pi}^*(\bar{Q}_t,\bar{A}_t)\,\mathrm{e}^{-r(t-\bar{T}_d)}dt - \mathrm{E}_{\bar{T}_d}\int_{\bar{T}_d}^{\infty}\hat{\pi}^*(Q_t,\boldsymbol{A})\,\mathrm{e}^{-r(t-\bar{T}_d)}dt)\\
&= \mathrm{E}_{0_+}\mathrm{e}^{-r\bar{T}_d}\,\bar{V}^*_{\bar{T}_d} - \mathrm{E}_{0_+}\mathrm{e}^{-r\bar{T}_d}\,V^*_{T,\bar{T}_d}
\end{aligned} \tag{A.8}$$

On the right-hand side of (A.8), using the formula for asset value in Goldstein et al. (2001), we derive the first term as

$$
\begin{aligned}
\mathrm{E}_{0_+}\mathrm{e}^{-r\bar{T}_d}\bar{V}^*_{\bar{T}_d} &= \mathrm{E}_{0_+}\mathrm{e}^{-r\bar{T}_d} A(\bar{Q}_{\bar{T}_d}, \bar{A}_{\bar{T}_d}) M(RD) \\
&= \bar{Q}^{1/\eta}_{\bar{T}_d} M(RD)\,\mathrm{E}_{0_+}\mathrm{e}^{-r\bar{T}_d}\,\bar{A}^{(\varepsilon-1)/\eta}_{\bar{T}_d} \\
&= A(\bar{Q}_{\bar{T}_d}, \boldsymbol{A}^{-}) M(RD)\,\mathrm{E}_{0_+}\mathrm{e}^{\{[\Lambda_\mu RD - 0.5((\Lambda_\sigma RD)^2 + (\rho\Xi_\sigma RD)^2)(1-(\varepsilon-1)\eta^{-1})](\varepsilon-1)\eta^{-1} - r\}\bar{T}_d} \\
&= \bar{J}(\boldsymbol{A}^{-}, \bar{\boldsymbol{Q}}_d(D)) \int_{0_+}^{\infty} \bar{H}_1(t)\, f_{0_+}^{\bar{T}_d}(t;D)\,dt,
\end{aligned}
$$

where

$$
\bar{J}(\boldsymbol{A}^{-}, \bar{\boldsymbol{Q}}_d(D)) \equiv A(\bar{Q}_{\bar{T}_d}, \boldsymbol{A}^{-}) M(RD), \quad \bar{\boldsymbol{Q}}_d(D) \equiv \bar{Q}_{\bar{T}_d} = D^{\eta}[(1-\tau)L]^{-\eta}\boldsymbol{A}^{1-\varepsilon},
$$

$$
\bar{H}_1(t) \equiv \mathrm{e}^{\{[\Lambda_\mu RD - 0.5((\Lambda_\sigma RD)^2 + (\rho\Xi_\sigma RD)^2)(1-(\varepsilon-1)\eta^{-1})](\varepsilon-1)\eta^{-1} - r\}t},
$$

$$
L \equiv \Phi\left[r - \frac{\mu_q + \Xi_\mu RD}{\eta} - \frac{(\sigma_q + \Xi_\sigma RD)^2}{2\eta} \times \frac{(1-\eta)}{\eta} \right]^{-1},
$$

$$
f_{0_+}^{\bar{T}_d}(t;D) \equiv \mathrm{n}\left(\frac{-\bar{z}(D) + \tilde{\sigma}_q^2(RD)\bar{x}\,t}{\tilde{\sigma}_q(RD)\sqrt{t}} \right) \frac{-\bar{z}(D)}{\tilde{\sigma}_q(RD)\,t^{3/2}}, \quad \bar{z}(D) \equiv \ln\frac{\bar{Q}_d(D)}{\bar{Q}}, \quad \bar{x} \equiv \tilde{\mu}_q(RD)\tilde{\sigma}_q^{-2}(RD) - 0.5,
$$

and $\mathrm{n}(\cdot)$ is the standard normal density function.

On the right-hand side of (A.8), the second term is equivalent to

$$
\mathrm{E}_{0_+}\mathrm{e}^{-r\bar{T}_d} V^*_{T,\bar{T}_d} = \mathrm{E}_{0_+}\mathrm{e}^{-r\bar{T}_d} A(Q_{\bar{T}_d}, \boldsymbol{A}) K(0).
$$

Because $\bar{Q}_d(\bar{T}_d) = \bar{Q}_d(0)\mathrm{e}^{(\mu_q + \Xi_\mu RD - 0.5(\sigma_q^2 + \Xi_\sigma^2 RD^2 + 2\sigma_q\Xi_\sigma RD))\bar{T}_d + (\sigma_q + \Xi_\sigma RD)B(\bar{T}_d)}$, $\bar{Q}_d(0) = Q_d(0)$, and $Q_d(\bar{T}_d) = Q_d(0)\mathrm{e}^{(\mu_q - 0.5\sigma_q^2)\bar{T}_d + \sigma_q B(\bar{T}_d)}$, we have

$$
\bar{Q}_d(\bar{T}_d) = Q_d(\bar{T}_d)\mathrm{e}^{(\Xi_\mu RD - 0.5(\Xi_\sigma^2 RD^2 + 2\sigma_q\Xi_\sigma RD))\bar{T}_d + (\Xi_\sigma RD)B(\bar{T}_d)},
$$

which implies $Q_d(\bar{T}_d) = \bar{Q}_d(\bar{T}_d)\mathrm{e}^{(-\Xi_\mu RD + 0.5(\Xi_\sigma^2 RD^2 + 2\sigma_q\Xi_\sigma RD))\bar{T}_d - (\Xi_\sigma RD)B(\bar{T}_d)}$. Plugging this into the second term in (A.8) and using Girsanov's theorem, we obtain:

$$
\begin{aligned}
\mathrm{E}_{0_+} e^{-r\bar{T}_d} V^*_{T,\bar{T}_d} &= \mathrm{E}_{0_+} A(\bar{Q}_{\bar{T}_d}, \boldsymbol{A}) K(0) e^{(-\Xi_\mu RD + 0.5(\Xi_\sigma^2 RD^2 + 2\sigma_q \Xi_\sigma RD))\eta^{-1}\bar{T}_d - r\bar{T}_d - (\Xi_\sigma RD)\eta^{-1} B(\bar{T}_d)} \\
&= \mathrm{E}_{0_+} \bar{V}^*_{T,\bar{T}_d} [1 - N(RD)] e^{(-\Xi_\mu RD + 0.5(\Xi_\sigma^2 RD^2 + 2\sigma_q \Xi_\sigma RD))\eta^{-1}\bar{T}_d - r\bar{T}_d - (\Xi_\sigma RD)\eta^{-1} B(\bar{T}_d)} \\
&= D(1-\tau)^{-1}(1 - N(RD)) \mathrm{E}_{0_+} e^{(-\Xi_\mu RD + 0.5(\Xi_\sigma^2 RD^2 + 2\sigma_q \Xi_\sigma RD))\eta^{-1}\bar{T}_d + 0.5\Xi_\sigma^2 RD^2 \eta^{-2}\bar{T}_d - r\bar{T}_d} \\
&= D(1-\tau)^{-1}(1 - N(RD)) \int_{0_+}^{\infty} \bar{H}_2(t) g_{0_+}^{\bar{T}_d}(t; D)\, dt
\end{aligned}
$$

where

$$
N(RD) \equiv \left[\frac{\Xi_\mu RD}{\eta} + \frac{0.5(1-\eta)(\Xi_\sigma^2 RD^2 + 2\sigma_q \Xi_\sigma RD)}{\eta^2} \right] K(0)^{-1},
$$

$$
g_{0_+}^{\bar{T}_d}(t; D) \equiv \mathrm{n}\left(\frac{-\bar{z}(D) + \tilde{\sigma}_q^2(RD)\bar{v}\, t}{\tilde{\sigma}_q(RD)\sqrt{t}} \right) \frac{-\bar{z}(D)}{\tilde{\sigma}_q(RD) t^{3/2}}; \quad \bar{v} \equiv (\tilde{\mu}_q(RD) - \tilde{\sigma}_q(RD)\Xi_\sigma RD \eta^{-1}) \tilde{\sigma}_q^{-2}(RD) - 0.5,
$$

and $\bar{H}_2(t) \equiv e^{\{[-\Xi_\mu RD + 0.5(\Xi_\sigma^2 RD^2 + 2\sigma_q \Xi_\sigma RD)]\eta^{-1} + 0.5\,\Xi_\sigma^2 RD^2 \eta^{-2} - r\}t}$. Hence, the formula for indirect bankruptcy costs due to inventor departures can be given by

$$
\bar{V}^{IBC} = \bar{J}(\boldsymbol{A}^-, \bar{\boldsymbol{Q}}_d(D)) \int_{0_+}^{\infty} \bar{H}_1(t) f_{0_+}^{\bar{T}_d}(t; D)\, dt - \bar{V}^*_{T,\bar{T}_d} [1 - N(RD)] \int_{0_+}^{\infty} \bar{H}_2(t) g_{0_+}^{\bar{T}_d}(t; D)\, dt \tag{A.9}
$$

Finally, regarding the total expected R&D expenses, we use the Feynman-Kac theorem and obtain:

$$
\bar{Q}\tilde{\mu}_q(RD)\bar{V}_{\bar{Q}}^{Expense} + 0.5\bar{Q}^2 \tilde{\sigma}_q^2(RD)\bar{V}_{\bar{Q}\bar{Q}}^{Expense} + RD = r\bar{V}^{Expense}
$$

$$
\textbf{s.t.} \quad \lim_{\bar{Q}\uparrow\infty} \bar{V}^{Expense}(\bar{Q}; D, RD) = RD\, r^{-1} \quad \text{and} \quad \lim_{\bar{Q}\downarrow\bar{Q}_d(D)} \bar{V}^{Expense}(\bar{Q}; D, RD) = 0\,.
$$

The explicit solution is given by

$$
\bar{V}^{Expense} = RD\, r^{-1}\left[1 - Q\,(\bar{\boldsymbol{Q}}, \bar{\boldsymbol{Q}}_d(D), \bar{x}, \bar{y}) \right] \tag{A.10}
$$

where $\bar{y} \equiv \frac{\sqrt{(\bar{x}\,\tilde{\sigma}_q^2(RD))^2 + 2r\,\tilde{\sigma}_q^2(RD)}}{\tilde{\sigma}_q^2(RD)}$. Using (A.7) - (A.10), we obtain the formula for available R&D benefit.

We now move our attention to the derivation of endogenous wages. Because the form of employment contract is identical to that in the baseline model, using the similar technique mentioned in Appendix B3, we obtain the following explicit form of endogenous wages:

$$\bar{I}^{*} = r\beta\bar{V}^{\,Performance}\,[1+(\varphi \mathrm{e}^{-rDu}-\varphi-1)\,Q\,(\bar{\boldsymbol{Q}},\bar{\boldsymbol{Q}}_d(D),\bar{x},\bar{y})\,]^{-1} \tag{A.11}$$

Using (A.7) - (A.11), we can further have the formula for before-tax ***net*** R&D value

$$\bar{V}^{\,RD} \equiv \bar{V}^{\,Performance} - \bar{V}^{\,Wage} = \bar{V}^{\,Performance} - \bar{I}^{*} r^{-1}\,[\,1-Q\,(\bar{\boldsymbol{Q}},\bar{\boldsymbol{Q}}_d(D),\bar{x},\bar{y})\,],$$

and the mixed-innovation version of the capital structure decision problem

$$\max{}_{D\geq 0}\,[\,(1-\tau)\,V_{T,0}^{*} + \bar{V}^{\,FL} + (1-\tau)\bar{V}^{\,RD}\,]$$

where $\bar{V}^{\,FL} = \tau\, d\, r^{-1} - (\tau\, d\, r^{-1} + \alpha\, D) Q\,(\bar{\boldsymbol{Q}},\bar{\boldsymbol{Q}}_d(D),\bar{x},\bar{y})$. This decision problem can be solved by invoking numerical techniques.

B.5. Solving the model allowing for inventor mobility

According to Section 6, the definition of the random time of inventor mobility will be $T_M := \inf\,(\,t>0 : Q_t \leq \boldsymbol{Q}_M(D,\theta)\,)$. After taking inventor mobility into account, we reformulate the expression of available R&D benefits as follows:

$$\tilde{V}^{\,Performance}(\boldsymbol{A}\;\;,\boldsymbol{Q};D,RD) \equiv \mathrm{E}_{0_+}\int_{0_+}^{T_M}[\,\hat{\pi}^{*}(Q_t,A_t)-\hat{\pi}^{*}(Q_t,\boldsymbol{A})-RD\,]\,\mathrm{e}^{-rt}\,dt\,.$$

This can be rearranged as

$$\begin{aligned}\tilde{V}^{\,Performance} &= V_I^{*} - \tilde{V}^{\,ILIM} - \tilde{V}^{\,Expense}\\ &\equiv \mathrm{E}_{0_+}\int_{0_+}^{\infty}[\,\hat{\pi}^{*}(Q_t,A_t)-\hat{\pi}^{*}(Q_t,\boldsymbol{A})\,]\,\mathrm{e}^{-rt}\,dt\\ &\quad -\mathrm{E}_{0_+}\mathrm{e}^{-rT_M}\,\mathrm{E}_{T_M}\int_{T_M}^{\infty}[\,\hat{\pi}^{*}(Q_t,A_t)-\hat{\pi}^{*}(Q_t,\boldsymbol{A})\,]\,\mathrm{e}^{-r(t-T_M)}dt - \mathrm{E}_{0_+}\int_{0_+}^{T_M} RD\,\mathrm{e}^{-rt}dt\end{aligned}$$

Note that the inclusion of inventor mobility does not affect the formula for intangible asset value, so we do not repeatedly derive it here. We put the focus on expected loss in intangible asset value due to inventor mobility $\tilde{V}^{ILIM}$ and total expected R&D expenses $\tilde{V}^{Expense}$.

The forms of the expected loss in intangible asset value due to inventors' mobility and of total expected R&D expenses are respectively similar to indirect bankruptcy costs and total expected R&D expenses in the baseline model (since we just replace T_d with T_M). Using the technique mentioned in Appendix B2, hence, we can derive their formulas as follows:

$$\tilde{V}^{ILIM} = J(\boldsymbol{A}\ , \theta \boldsymbol{Q}_d(D)) \int_{0_+}^{\infty} H(t)\, f_{0_+}^{T_M}(t;D)\, dt - V_{T,T_M}^{*}\, Q\,(\boldsymbol{Q}, \theta \boldsymbol{Q}_d(D), x, y) \tag{A.11}$$

$$\tilde{V}^{Expense} = RD\, r^{-1} - RD\, r^{-1} Q\,(\boldsymbol{Q}, \theta \boldsymbol{Q}_d(D), x, y) \tag{A.12}$$

where $f_{0_+}^{T_M}(t;D) \equiv \mathrm{n}\left(\frac{\sigma_q^2 x t - z(D) - \ln\theta}{\sigma_q \sqrt{t}} \right) \frac{-z(D) - \ln\theta}{\sigma_q \sqrt{t^3}}$. Using (A.2), (A.11), and (A.12), we obtain the formula for available R&D benefit.

Next, we consider the derivation for endogenous wages. Recall from Section 6 that, in the model allowing for inventor mobility, inventors' collective departures are spontaneous and not triggered by the firm's bankruptcy. Hence, when specifying the net present value of an employment contract, unemployment costs should be excluded. This means that $\tilde{V}^{Pay} \equiv \mathrm{E}_0 \int_0^{T_M} \tilde{I}^{*} \mathrm{e}^{-rs} ds = \tilde{V}^{Wage}$ where endogenous wages will be $\tilde{I}^{*} = r\beta \tilde{V}^{Performance} [1 - Q\,(\boldsymbol{Q}, \theta \boldsymbol{Q}_d(D), x, y)]^{-1}$. The capital structure decision problem has a corresponding version: $\max_{D \geq 0} [(1-\tau) V_{T,0}^{*} + (1-\tau)\tilde{V}^{RD} + V^{FL}]$. This completes the proof.

B.6. Solving the model allowing for human capital risk management

At the beginning of this section, we solve the size of upward wage adjustment endogenously. Recall that at each of the wage-adjustment time points, managers reset wages according to their competitive level. Therefore, just after the wage-adjustment time point, the continuation value of wage contracts should equal the value of laborers' outside option. Besides, managers adjust wages so long as the ratio of laborers' continuation value to outside option value touches the lower bound $1-\vartheta$. Such two ideas jointly imply: for $\forall i \in \mathbb{N}$, $V^{CON}(Q_{Tw^i};D,I_{i-1})(1-\vartheta)^{-1} = \boldsymbol{S} = V^{CON}(Q_{Tw_+^i};D,I_i)$. This helps us derive the size of wage adjustment I_i / I_{i-1} from the following:

$$\mathrm{E}_{Tw^i}\int_{Tw^i}^{T_d} \tfrac{I_{i-1}}{1-\vartheta}\, \mathrm{e}^{-r(s-Tw^i)} ds = \frac{V^{CON}(Q_{Tw^i};D,I_{i-1})}{1-\vartheta} = V^{CON}(Q_{Tw_+^i};D,I_i) = \mathrm{E}_{Tw_+^i}\int_{Tw_+^i}^{T_d} I_i\, \mathrm{e}^{-r(s-Tw_+^i)} ds\,.$$

Because the evolution of product demand is governed by a continuous-time Brownian motion and $Q_{Tw^i} \approx Q_{Tw_+^i}$, we have $I_i \approx \frac{I_{i-1}}{1-\vartheta}$, which implies $\frac{I_i}{I_{i-1}} = \frac{1}{1-\vartheta} \equiv \phi^{-1}$.

We are ready to solve the explicit form of the costs of fully hedging human capital risk due to inventor mobility. This follows

$$\begin{aligned} V^{HIM} &\equiv \mathrm{E}_{0+}\sum_{i=1}^{\infty} [V^{CON}(Q_{Tw_+^i};D,I_i) - V^{CON}(Q_{Tw^i};D,I_{i-1})]\, \mathrm{e}^{-r\,Tw^i} \\ &\equiv \mathrm{E}_{0+}\sum_{i=1}^{\infty} V_i^{HC}(Q_{Tw^i};D)\, \mathrm{e}^{-r\,Tw^i} \end{aligned}$$

where $Tw^i := \inf\,(t>0 : Q_t \le \boldsymbol{Q}_W^i(D,\phi))$. Hence,

$$\begin{aligned} V_i^{HC}(Q_{Tw^i};D) &= \mathrm{E}_{Tw_+^i}\int_{Tw_+^i}^{T_d} I_i\, \mathrm{e}^{-r(s-Tw_+^i)} ds - \mathrm{E}_{Tw^i}\int_{Tw^i}^{T_d} I_{i-1}\, \mathrm{e}^{-r(s-Tw^i)} ds \\ &\approx \mathrm{E}_{Tw^i}\int_{Tw^i}^{T_d} (I_i - I_{i-1})\, \mathrm{e}^{-r(s-Tw^i)} ds, \end{aligned}$$

which satisfies the below ODE (let $I_i = I_0\,\phi^{-i}$, $V_{i,Q}^{HC} \equiv \frac{\partial V_i^{HC}}{\partial Q}$, and $V_{i,QQ}^{HC} \equiv \frac{\partial^2 V_i^{HC}}{\partial Q^2}$)

$$V_{i,Q}^{HC}\, Q\,\mu_q + 0.5\, Q^2 \sigma_q^2 V_{i,QQ}^{HC} + I_0(\phi^{-i} - \phi^{1-i}) - r V_i^{HC} = 0$$

$$\textbf{s.t.}\ \lim_{Q\uparrow\infty} V_i^{HC}(Q;D) = I_0(\phi^{-i} - \phi^{1-i})\, r^{-1};\ \text{and}\ \lim_{Q\downarrow Q_d(D)} V_i^{HC}(Q;D) = 0\,.$$

Then we have $V_i^{HC}(Q_{Tw^i};D) = I_0(\phi^{-i} - \phi^{1-i})\, r^{-1}[1 - Q\,(\boldsymbol{Q}_W^i(D,\phi), \boldsymbol{Q}_d(D), x, y)]$ and rewrite hedging costs as

$$V^{HIM} \equiv \mathrm{E}_{0+} \sum_{i=1}^{\infty} V_i^{HC}(Q_{Tw^i}; D)\, \mathrm{e}^{-r\,Tw^i} = \sum_{i=1}^{\infty} \mathrm{E}_{0+} V_i^{HC}(Q_{Tw^i}; D)\, \mathrm{e}^{-r\,Tw^i}$$

Let $V_i^{\mathbf{HC}} \equiv \mathrm{E}_{0+} V_i^{HC}(Q_{Tw^i}; D)\, \mathrm{e}^{-r\,Tw^i}$. This solves the below ODE:

$$V_{i,Q}^{\mathbf{HC}}\, Q\, \mu_q + 0.5\, Q^2 \sigma_q^2 V_{i,QQ}^{\mathbf{HC}} - r V_i^{\mathbf{HC}} = 0$$

s.t. $\lim_{Q\uparrow\infty} V_i^{\mathbf{HC}}(Q; D) = 0$; and $\lim_{Q\downarrow Q_W^i(D,\phi)} V_i^{\mathbf{HC}}(Q; D) = V_i^{HC}(Q_{Tw^i}; D)$.

Hence, expected hedging costs at the *i*th wage-adjustment time point take a form:

$$\begin{aligned} V_i^{\mathbf{HC}} &= V_i^{HC}(Q_{Tw^i}; D)\, Q\,(\boldsymbol{Q}, \boldsymbol{Q}_W^i(D,\phi), x, y) \\ &= I_0(\phi^{-i} - \phi^{1-i})\, r^{-1} [1 - Q\,(\boldsymbol{Q}_W^i(D,\phi), \boldsymbol{Q}_d(D), x, y)]\, Q\,(\boldsymbol{Q}, \boldsymbol{Q}_W^i(D,\phi), x, y) \\ &= I_0(\phi^{-i} - \phi^{1-i})\, r^{-1} [Q\,(\boldsymbol{Q}, \boldsymbol{Q}_W^i(D,\phi), x, y) - Q\,(\boldsymbol{Q}, \boldsymbol{Q}_d(D), x, y)] \end{aligned}$$

and the explicit form of total hedging costs is given by

$$V^{HIM} = \sum_{i=1}^{\infty} I_0(\phi^{-i} - \phi^{1-i})\, r^{-1} [Q\,(\boldsymbol{Q}, \boldsymbol{Q}_W^i(D,\phi), x, y) - Q\,(\boldsymbol{Q}, \boldsymbol{Q}_d(D), x, y)]$$

where I_0 denotes initial endogenous wages that satisfy the condition (similar to $\tilde{I}^*$):

$$\beta V^{Performance} = V^{CON}(\boldsymbol{Q}; D, I_0) = \mathrm{E}_0 \int_0^{T_d} I_0\, \mathrm{e}^{-rs}\, ds\,.$$

This implies $I_0 = r\beta V^{Performance} [1 - Q\,(\boldsymbol{Q}, \boldsymbol{Q}_d(D), x, y)]^{-1}$.

Combining hedging costs with (A.1), we obtain the formula for (before-tax) net R&D value under human capital risk management $\hat{V}^{RD} = (1-\beta)\, V^{Performance} - V^{HIM}$. Then the corresponding capital structure decision problem will become

$$\max_{D \geq 0} [(1-\tau) V_{T,0}^* + (1-\tau)\hat{V}^{RD} + V^{FL}].$$

We solve optimal debt policy under human capital risk management numerically using this modified capital structure decision problem.

B.7. Solving the model allowing for limited innovation failure tolerance

Under limited innovation failure tolerance, the expression of available R&D benefit can be shown below:

$$\bar{\bar{V}}^{Performance}(\boldsymbol{A}\ ,\boldsymbol{Q};D,RD)\equiv \mathrm{E}_{0_+}\int_{0_+}^{T_d\wedge \boldsymbol{T_t}}[\hat{\pi}^*(Q_t,A_t)-\hat{\pi}^*(Q_t,\boldsymbol{A})-RD]\,\mathrm{e}^{-rt}\,dt$$

where $\boldsymbol{T_t}:=\inf\,(t>0:A_t\le \boldsymbol{A_t})$.[35] This expression can be rearranged as

$$\begin{aligned}\bar{\bar{V}}^{Performance} &= V_I^* - \bar{\bar{V}}^{Expense} - \bar{\bar{V}}^{IBC}\\ &\equiv \mathrm{E}_{0_+}\int_{0_+}^{\infty}[\hat{\pi}^*(Q_t,A_t)-\hat{\pi}^*(Q_t,\boldsymbol{A})]\,\mathrm{e}^{-rt}\,dt - \mathrm{E}_{0_+}\int_{0_+}^{T_d\wedge \boldsymbol{T_t}} RD\,\mathrm{e}^{-rt}\,dt\\ &\quad - \mathrm{E}_{0_+}\mathrm{e}^{-rT_d\wedge \boldsymbol{T_t}}\,\mathrm{E}_{T_d\wedge \boldsymbol{T_t}}\int_{T_d\wedge \boldsymbol{T_t}}^{\infty}[\hat{\pi}^*(Q_t,A_t)-\hat{\pi}^*(Q_t,\boldsymbol{A})]\,\mathrm{e}^{-r(t-T_d\wedge \boldsymbol{T_t})}\,dt\end{aligned}$$

Note that the inclusion of limited tolerance towards innovation failure does not affect the formula for intangible asset value, so we do not repeatedly derive it here. We put the focus on indirect bankruptcy costs and total expected R&D expenses.

Consider total expected R&D expenses under limited innovation failure tolerance:

$$\bar{\bar{V}}^{Expense} = \mathrm{E}_{0_+}\int_{0_+}^{T_d\wedge \boldsymbol{T_t}} RD\,\mathrm{e}^{-rt}\,dt\,.$$

Because, for any time $s>0$, $\{\omega:T_d(\omega)>s\}\cap\{\omega:T_d(\omega)<\boldsymbol{T_t}(\omega)\}\subset\{\omega:\boldsymbol{T_t}(\omega)>s\}$ and $\{\omega:\boldsymbol{T_t}(\omega)>s\}\cap\{\omega:\boldsymbol{T_t}(\omega)<T_d(\omega)\}\subset\{\omega:T_d(\omega)>s\}$, it is known that T_d and $\boldsymbol{T_t}$ are two independent random times, which follow

[35] Limited tolerance toward innovation failure is modeled using managers' option to abrogate inventors' employment contract. If technology innovation failure is too serious (the productivity technology level falls below the tolerance threshold $\boldsymbol{A_t}$), managers will fire inventors in place and immediately suspend R&D investment. We assume that managers do not recruit new inventors after old ones leave for brevity. Managers endogenously choose the tolerance threshold that satisfies the condition of "zero" remaining intangible asset value $V^*_{I,\boldsymbol{T_t}}=0$.

$$
\begin{aligned}
1_{(\omega:T_d(\omega)\wedge T_t(\omega)>s)} &= 1_{(\omega:T_d(\omega)>s)}1_{(\omega:T_d(\omega)<T_t(\omega))} + 1_{(\omega:T_t(\omega)>s)}1_{(\omega:T_d(\omega)>T_t(\omega))} \\
&= 1_{(\omega:T_d(\omega)>s)}1_{(\omega:T_d(\omega)<T_t(\omega))}1_{(\omega:T_t(\omega)>s)} + 1_{(\omega:T_t(\omega)>s)}1_{(\omega:T_d(\omega)>T_t(\omega))}1_{(\omega:T_d(\omega)>s)} \\
&= 1_{(\omega:T_d(\omega)>s)}1_{(\omega:T_t(\omega)>s)}(1_{(\omega:T_d(\omega)<T_t(\omega))} + 1_{(\omega:T_d(\omega)>T_t(\omega))}) \\
&= 1_{(\omega:T_d(\omega)>s)}1_{(\omega:T_t(\omega)>s)}
\end{aligned}
$$

Given $F_{0_+}^{T_t}(s;D) \equiv \mathrm{Q}(T_t > s \mid \mathrm{F}_0)$ and $F_{0_+}^{T_d}(s;D) \equiv \mathrm{Q}(T_d > s \mid \mathrm{F}_0)$, hence, we have

$$
\begin{aligned}
\overline{\overline{V}}^{\,Expense} &= \mathrm{E}_0 \int_0^\infty RD\,\mathrm{e}^{-rs} 1_{(\omega:T_d(\omega)\wedge T_t(\omega)>s)}\,ds = \int_0^\infty RD\,\mathrm{e}^{-rs}\,\mathrm{E}_0\,(1_{(\omega:T_d(\omega)>s)}1_{(\omega:T_t(\omega)>s)})\,ds \\
&= \int_0^\infty RD\,\mathrm{e}^{-rs}\,\mathrm{Q}(T_d > s \mid \mathrm{F}_0)\,\mathrm{Q}(T_t > s \mid \mathrm{F}_0)\,ds \qquad \text{(A.13)}
\end{aligned}
$$

Next, we consider the indirect bankruptcy cost under limited innovation failure tolerance. Because $V_{I,T_t}^* = 0$, we have $\overline{\overline{V}}^{\,IBC} \equiv \mathrm{E}_0 V_{I,T_d}^*\,\mathrm{e}^{-rT_d} 1_{(T_d < T_t)}$. In the beginning, we need to prepare several lemmas that describe the law of the joint probability of productivity technology at default, random default time, and random inventor-dismissal time. Applying Theorem 3.20 in Klebaner (2005) and Theorem A2 in Chen et al. (2011), we obtain:

$$
\begin{aligned}
\mathrm{Q}_0(\inf_{0<s<t} A_s = y, A_t = z) &= \frac{\sqrt{2}\ln(z\boldsymbol{A}\ /y^2)}{\sqrt{\pi}\,t^{3/2}\sigma_A^3(RD)zy}\exp\left\{\frac{[2\ln y - \ln z - \ln\boldsymbol{A}\ + v\sigma_A^2(RD)t]^2}{-2\sigma_A^2(RD)t}\right\}\left(\frac{y}{\boldsymbol{A}}\right)^{2v} \\
&= f_{\mathrm{A},T_t}(y,z,t)
\end{aligned}
$$

where $0 \le t \le \infty$, $\boldsymbol{A}_t \le z \le \infty$, $\boldsymbol{A}_t \le y \le z$, and $v \equiv \mu_A(RD)\sigma_A^{-2}(RD) - 0.5$. So, $\overline{\overline{V}}^{\,IBC}$ is given by

$$
\begin{aligned}
\overline{\overline{V}}^{\,IBC} &= \mathrm{E}_0\left[\,\mathrm{E}\left[V_I^*(A_{T_d}, \boldsymbol{Q}_d(D); RD)\,\mathrm{e}^{-rT_d}1_{(T_d<T_t)} \,\middle|\, \mathrm{F}_{T_d}^{\,B} \vee \mathrm{F}_0^{\,W}\right]\right] \\
&= \mathrm{E}_0 \int_{\boldsymbol{A}_t}^\infty \int_{\boldsymbol{A}_t}^z V_I^*(z, \boldsymbol{Q}_d(D); RD)\,\mathrm{e}^{-rT_d} f_{\mathrm{A},T_t}(y,z,T_d)\,dy\,dz
\end{aligned}
$$

Because the density function of default time is $f_{0_+}^{T_d}(t;D) \equiv \mathrm{n}\left(\frac{-z(D)+\sigma_q^2 xt}{\sigma_q\sqrt{t}}\right)\frac{-z(D)}{\sigma_q\sqrt{t^3}}$, we have

$$
\overline{\overline{V}}^{\,IBC} = \int_{0_+}^\infty \int_{\boldsymbol{A}_t}^\infty \int_{\boldsymbol{A}_t}^z V_I^*(z, \boldsymbol{Q}_d(D); RD)\, f_{\mathrm{A},T_t}(y,z,t)\, f_{0_+}^{T_d}(t;D)\,\mathrm{e}^{-rt}\,dy\,dz\,dt \qquad \text{(A.14)}
$$

where

$$F_{0_+}^{T_d}(t;D) \equiv \mathrm{N}\left(\tfrac{-z(D)+\sigma_q^2 xt}{\sigma_q\sqrt{t}}\right) - \mathrm{N}\left(\tfrac{z(D)+\sigma_q^2 xt}{\sigma_q\sqrt{t}}\right) Q\,(\boldsymbol{Q}_d(D), \boldsymbol{Q}, -x, -x)\,,$$

$$F_{0_+}^{T_t}(t;D) \equiv \mathrm{N}\left(\tfrac{\ln \boldsymbol{A} \;-\ln \boldsymbol{A}_t + (\sigma_A(RD))^2 vt}{\sigma_A(RD)\sqrt{t}}\right) - \mathrm{N}\left(\tfrac{\ln \boldsymbol{A}_t - \ln \boldsymbol{A} \;+ (\sigma_A(RD))^2 vt}{\sigma_A(RD)\sqrt{t}}\right) Q\,(\boldsymbol{A}_t, \boldsymbol{A} \;, -v, -v)\,,$$

and $\mathrm{N}(\cdot)$ is the standard normal probability function. Using (A.2), (A.13), and (A.14), we obtain the formula for available R&D benefit under limited innovation failure tolerance.

Similar to our baseline model, the net present value of the employment contract is $\bar{\bar{V}}^{Pay} \equiv \mathrm{E}_{0_+}\left[\int_t^{T_d \wedge T_t} \bar{\bar{I}}\, \mathrm{e}^{-rt} dt - \mathrm{e}^{-r\, T_d \wedge T_t} \int_{T_d \wedge T_t}^{T_d \wedge T_t + Du} \varphi \bar{\bar{I}}\, \mathrm{e}^{-r(t - T_d \wedge T_t)} dt\right]$. Similar to (A.13), we derive endogenous wages as follows:

$$\bar{\bar{I}}^{*} = \beta\, \bar{\bar{V}}^{Performance} \left\{ \int_0^{\infty} \mathrm{e}^{-rs}\, \Delta(s)\, ds + r^{-1} \varphi\,(\mathrm{e}^{-rDu} - 1) \int_0^{\infty} \mathrm{e}^{-rs}\, [\Upsilon(s) + \Psi(s)]\, ds \right\}^{-1}$$

where

$$\Delta(s) \equiv F_{0_+}^{T_d}(s;D) F_{0_+}^{T_t}(s;D)\,, \;\; \Upsilon(s) \equiv f_{0_+}^{T_d}(s;D) F_{0_+}^{T_t}(s;D)\,, \;\; \Psi(s) \equiv f_{0_+}^{T_t}(s;D) F_{0_+}^{T_d}(s;D)\,,$$

$$f_{0_+}^{T_t}(s;D) \equiv \mathrm{n}\left(\frac{\ln \boldsymbol{A} \;-\ln \boldsymbol{A}_t + (\sigma_A(RD))^2 vs}{\sigma_A(RD)\sqrt{s}} \right) \frac{\ln \boldsymbol{A} \;-\ln \boldsymbol{A}_t}{\sigma_A(RD)\sqrt{s^3}}\,.$$

Hence, net (before-tax) R&D value earned by shareholders correspondingly equals

$$\bar{\bar{V}}^{RD} = \bar{\bar{V}}^{Performance} - \int_0^{\infty} \bar{\bar{I}}^{*} \mathrm{e}^{-rs}\, \Delta(s)\, ds\,.$$

Replacing V^{RD} in decision problem (8) with $\bar{\bar{V}}^{RD}$, we obtain the decision problem of the capital structure under limited innovation failure tolerance:

$$\max_{D \ge 0}\; [(1-\tau) V_{T,0}^{*} + (1-\tau)\, \bar{\bar{V}}^{RD} + V^{FL}\;].$$

The optimal debt choice can be numerically solved from this modified decision problem.

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